

Experimentalphysik IV (PEP4)

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Übung 9

Abgabe am Montag, den 16.6.2014 vor der Vorlesung

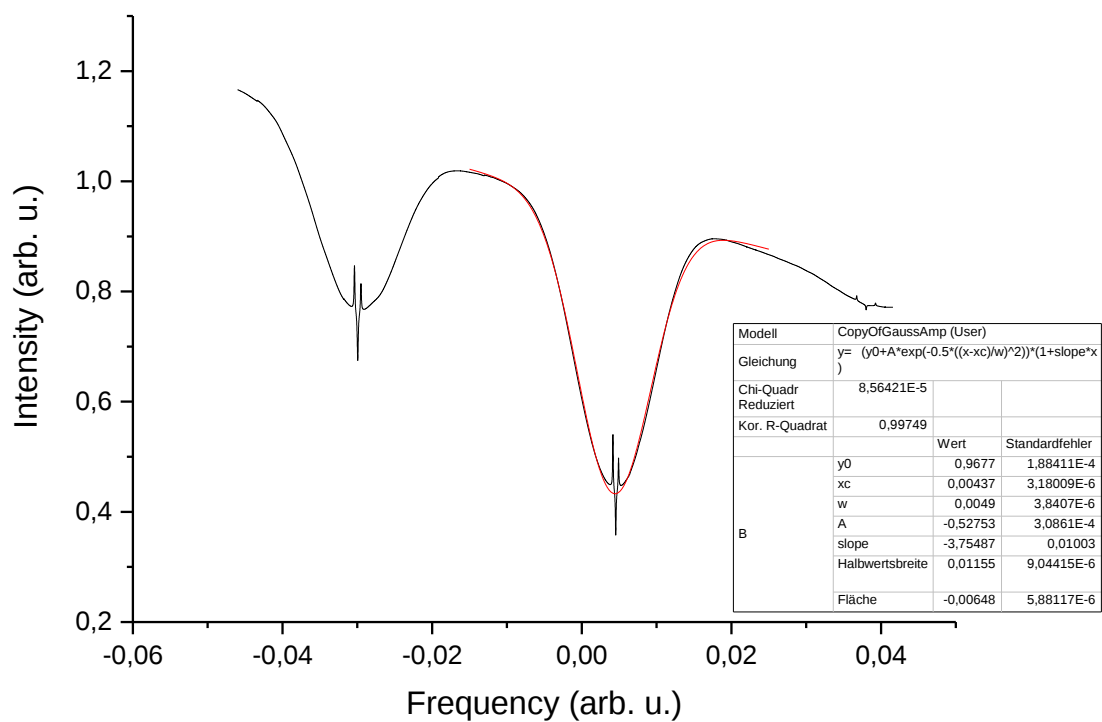
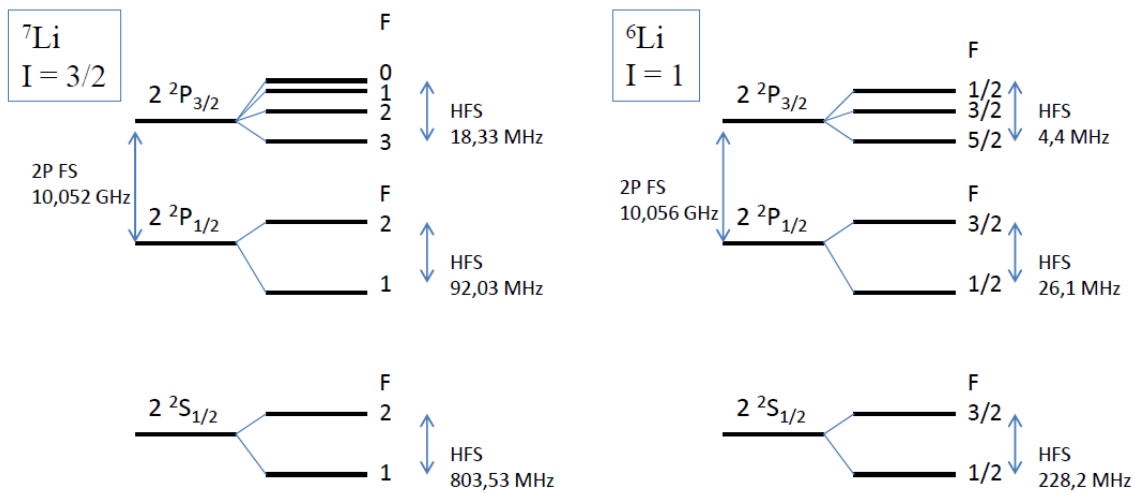
Aufgabe 1: Dopplerfreie Spektroskopie an Lithium (6P)

In der Vorlesung wurde der Übergang $2s \rightarrow 2p$ bei Lithium mittels dopplerfreier Spektroskopie untersucht. Es wurde dabei isotonenangereichertes 6Li verwendet, das einen kleinen Anteil von 7Li enthält. Unten finden Sie die Niveauschemata von 6Li und 7Li . Das gemessene Absorptionsspektrum soll analysiert werden.

Benutzen Sie dazu die auf Moodle bereitgestellten ASCII Daten des Spektrums, die Sie mit Ihrem eigenen Plotprogramm darstellen können. Dabei enthält die erste Spalte die Frequenz (beliebige Einheiten) und die zweite Spalte die mit einer Photodiode gemessene Intensität (beliebige Einheiten).

Im Diagramm sehen Sie die dopplerverbreiterten Linien von 6Li . Ihre Wellenlängen wurden in der Vorlesung zu $670,99144\text{ nm}$ (D1) und $670,97634\text{ nm}$ (D2) bestimmt. Auf diesen breiten Linien sind scharfe, dopplerfrei gemessene Linien zu sehen, von denen die nach oben zeigenden Spitzen Übergänge zwischen verschiedenen Hyperfeinzuständen sind. (Ignorieren Sie die nach unten zeigenden Spitzen.) Am rechten Ende des Spektrums finden Sie schwache Linien, die zur D2 Linie von 7Li gehören.

- Kalibrieren Sie die Frequenzachse mit den gemessenen Wellenlängen der D1 und D2 Linien. Welchen Übergängen entsprechen die D1- und D2-Linien in dem Energieschema? Welche Hyperfeinzustände sind in den Übergängen der feiner aufgelösten Spitzen involviert? Beachten Sie, dass für einen Teil der Zustände die Aufspaltung der Hyperfeinstruktur so klein ist, dass sie in unserem Experiment nicht aufgelöst werden kann. Vergleichen Sie die gemessenen Linienabstände mit denen der Energieschemata.
- Schätzen Sie die Temperatur des Lithiumgases anhand der Linienbreiten ab.
- Bestimmen Sie die Isotopieverschiebung von 6Li bezüglich 7Li . Schätzen Sie den Beitrag der unterschiedlichen reduzierten Massen auf die Isotopieverschiebung ab.



Lösung:

- Informationen zur dopplerfreien Sättigungsspektroskopie am Ende dieser Musterlösung (Anhang B).
- Achtung: hier wurden die Positionen der D-Linien näherungsweise durch die Mittenfrequenz zwischen den beiden Hyperfeinlinien angenommen. Eine genauere Positionsbestimmung mit Hilfe beider Hyperfeinlinien ist am Ende dieser Lösung zu finden (Anhang A).

a) Die Hyperfeinstruktur der angeregten p-Zustände wird vernachlässigt, da sie nicht aufgelöst werden. Die Auflösung ist < 100 MHz.

$$D1: 2^2S_{1/2} \rightarrow 2^2P_{1/2}$$

$$D2: 2^2S_{1/2} \rightarrow 2^2P_{3/2}$$

Kalibration der Frequenzachse:

$${}^6\text{Li}: D1: f1 = -0.02994 \quad \text{Gemessen: } \lambda1(D1) = 670,99144 \text{ nm} \quad \nu1 = c/\lambda1 = 446.79 \text{ THz}$$

$${}^6\text{Li}: D2: f2 = 0.004565 \quad \lambda2(D2) = 670,97634 \text{ nm} \quad \nu2 = c/\lambda2 = 446.8 \text{ THz}$$

$${}^7\text{Li}: D2: f1 = 0.038$$

$$\text{Kalibrationsfaktor } k = \Delta\nu/(f2-f1) = 10.056 \text{ GHz}/0.034505 = 291.44 \text{ GHz/unit}$$

$${}^6\text{Li}: \text{ Hyperfinelinien } -0.03038 \text{ und } -0.02951 \text{ oder } 0.00418 \text{ und } 0.004917$$

$$\Delta\nu_{HF}^6 = k(-0.03038 - (-0.02951)) = 253 \text{ MHz} \quad (\text{Energieschema } 228 \text{ MHz})$$

$$\Delta\nu_{HF}^6 = k(0.004917 - 0.00418) = 215 \text{ MHz} \quad (\text{Energieschema } 228 \text{ MHz})$$

$${}^7\text{Li}: \text{ Hyperfinelinien } 0.03674 \text{ und } 0.03927$$

$$\Delta\nu_{HF}^7 = k(0.03927 - 0.03674) = 737 \text{ MHz} \quad (\text{Energieschema } 803 \text{ MHz})$$

Diese Frequenzabstände entsprechen näherungsweise den Hyperfeinaufspaltungen der $2^2S_{1/2}$ Grundzustände.

Folgende Übergänge werden beobachtet:

$${}^6\text{Li}, D1: 2^2S_{\frac{1}{2}}(F = \frac{1}{2}, \frac{3}{2}) \rightarrow 2^2P_{\frac{1}{2}}(F = \frac{1}{2}, \frac{3}{2})$$

$${}^6\text{Li}, D2: 2^2S_{\frac{1}{2}}(F = \frac{1}{2}) \rightarrow 2^2P_{\frac{3}{2}}(F = \frac{1}{2}, \frac{3}{2})$$

$$2^2S_{\frac{1}{2}}(F = \frac{3}{2}) \rightarrow 2^2P_{\frac{3}{2}}(F = \frac{1}{2}, \frac{3}{2}, \frac{5}{2})$$

$${}^7\text{Li}, D2: 2^2S_{\frac{1}{2}}(F = 1) \rightarrow 2^2P_{\frac{3}{2}}(F = 0, 1, 2)$$

$$2^2S_{\frac{1}{2}}(F = 2) \rightarrow 2^2P_{\frac{3}{2}}(F = 1, 2, 3)$$

b)

$$\text{Dopplereffekt durch die Geschwindigkeitskomponente in z-Richtung: } \Delta\nu = \nu_L \frac{v_z}{c}$$

$$\text{Geschwindigkeitsverteilung gegeben durch } p(v_z) \propto \exp\left(-\frac{E_{kin}}{k_B T}\right) = \exp\left(-\frac{\frac{1}{2} m v_z^2}{k_B T}\right)$$

Bem.: nicht die Maxwell-Boltzmann-Verteilung (diese hat den Faktor v^2 zusätzlich), da wir nicht $p(|\vec{v}|)$ suchen.

Mit der Annahme einer in $p(v_z)$ linearen Absorption und dem Dopplereffekt

$$\text{ergibt sich: } Absorption \propto \exp\left(-\frac{\frac{1}{2} m \left(\frac{\Delta\nu}{\nu_L} c\right)^2}{k_B T}\right)$$

$$\text{Die Breite aus dem Gauß: } \exp\left(-\frac{\Delta\nu^2}{2\sigma^2}\right) \quad \text{ist: } \sigma = \sqrt{\frac{k_B T}{c^2 m}} \nu_L \rightarrow T = \left(\frac{\sigma}{\nu_L}\right)^2 \frac{c^2 m}{k_B}$$

$$\sigma \sim 0.0049 * k = 1.42 \text{ GHz} \rightarrow T \sim 385^\circ\text{C}, \text{ (siehe fit im Graphen ...)}$$

Die tatsächliche Temperatur war ca. 345°C. Abweichung? Vermutlich verbreitert die kaum sichtbare 7Li D1-Linie, die mit der 6Li D2 Linie überlagert ist, das Signal etwas?

c)

c) Rydberg Formel mit Quantendefekt

$$E_{n,l} = -E_{\text{Ryd}} \frac{1}{(n - \Delta_{n,l})^2} = -\frac{\mu e^4}{8\epsilon_0^2 h^2} \cdot \frac{1}{(n - \Delta_{n,l})^2}$$

reduzierte Masse

Hier Übergang $n=2, l=0 \rightarrow n=2, l=1$

$$\begin{aligned} \Delta E (n=2, l=0 \rightarrow n=2, l=1) &= \\ &= -\frac{\mu e^4}{8\epsilon_0^2 h^2} \left(\frac{1}{2 - \Delta_{2,1}} - \frac{1}{2 - \Delta_{2,0}} \right) \end{aligned}$$

Die Energie des Übergangs skaliert hier also mit der reduzierten Masse μ .

$$\Rightarrow \frac{\Delta E_{\text{Li7}}}{\Delta E_{\text{Li6}}} = \frac{\mu_7}{\mu_6} \Rightarrow (\Delta E_{\text{Li7}} - \Delta E_{\text{Li6}}) = \left(\frac{\mu_7}{\mu_6} - 1 \right) \Delta E_{\text{Li6}}$$

$$(\Delta E_{\text{Li7}} - \Delta E_{\text{Li6}}) = \left(\frac{\frac{m_e}{m_{\text{Li6}}} + 1}{\frac{m_e}{m_{\text{Li7}}} + 1} - 1 \right) \Delta E_{\text{Li6}}$$

$$\Delta E_{\text{Li6}} = 4,468 \cdot 10^{14} \text{ Hz}$$

$$(\Delta E_{\text{Li7}} - \Delta E_{\text{Li6}}) = 1,297 \cdot 10^{-5} \cdot \Delta E_{\text{Li6}} = \underline{\underline{5,8 \text{ GHz}}}$$

$$\text{Gemessen: } D2(\text{Li7}) - D2(\text{Li6}) = 9,86 \text{ GHz}$$

Weitere Effekte: Kernvolumeneffekt.

Anhang A

Genauere Positionsbestimmung der D-Linien:

$$\Delta E_{\text{HFS}} = A \left(F(F+1) - j(j+1) - I(I+1) \right)$$

$$\boxed{{}^6\text{Li}} \quad I=1, \quad \text{zs: } j = \frac{1}{2}$$

$$\begin{aligned} \Delta E_{\text{HFS}} &= A \left(F(F+1) - \frac{3}{4} - 2 \right) = \\ &= A \left(F(F+1) - \frac{11}{4} \right) \end{aligned}$$

$$\left. \begin{array}{l} F = \frac{1}{2} \quad \Delta E_{\text{HFS}} = A \left(-\frac{8}{4} \right) = -2A \\ F = \frac{3}{2} \quad \Delta E_{\text{HFS}} = A \frac{4}{4} = +A \end{array} \right\} \text{Abstand } 3A$$

$$\begin{aligned} \boxed{{}^7\text{Li}} \quad \Delta E_{\text{HFS}} &= A \left(F(F+1) - \frac{3}{4} - \frac{15}{4} \right) \\ &= A \left(F(F+1) - \frac{9}{2} \right) \end{aligned}$$

$$\left. \begin{array}{l} F=1 \quad \Delta E_{\text{HFS}} = A \left(-\frac{5}{2} \right) = -\frac{5}{2}A \\ F=2 \quad \Delta E_{\text{HFS}} = A \frac{3}{2} = \frac{3}{2}A \end{array} \right\} \text{Abstand } 4A$$

Position der D-Linien bei ${}^6\text{Li}$: $\frac{A}{3A} = \frac{1}{3}$ des Abstandes der HF-Linien oberhalb der intensiveren $F=\frac{3}{2}$ Linie

${}^7\text{Li}$: $\frac{\frac{3}{2}A}{4A} = \frac{3}{8}$ des Abstandes der HF-Linien oberhalb der intensiveren $F=2$ Linie.

8.3.1 Principle of saturated absorption spectroscopy

This method of laser spectroscopy exploits the saturation of absorption to give a Doppler-free signal. At high intensities the population difference between two levels is reduced as atoms are excited to the upper level, and we account for this by modifying eqn 8.11 to read

$$\kappa(\omega) = \int_{-\infty}^{\infty} \{N_1(v) - N_2(v)\} \sigma_{\text{abs}}(\omega - kv) dv. \quad (8.17)$$

This is the same as the modification we made in going from eqn 7.70 to 7.72 but applied to each velocity class within the distribution. Here $N_1(v)$ and $N_2(v)$ are the number densities in levels 1 and 2, respectively, for atoms with velocities between v and $v + dv$. At low intensities almost all the atoms stay in level 1, so $N_1(v) \simeq N(v)$ has the Gaussian distribution in eqn 8.3 and $N_2 \simeq 0$, as illustrated in Fig. 8.3(a). For all intensities, the integral of the number densities in each velocity class equals the total number density in that level, i.e.

$$\int_{-\infty}^{\infty} N_1(v) dv = N_1, \quad (8.18)$$

and similarly for N_2 . The total number density $N = N_1 + N_2$.¹⁸

In saturated absorption spectroscopy the quantity $N_1(v) - N_2(v)$ is affected by interaction with a strong laser beam, as shown in Fig. 8.3(b) and Fig. 8.4 shows a typical experimental arrangement. The beam splitter divides the power of the laser beam between a weak probe and a stronger pump beam.¹⁹ Both these beams have the same frequency ω and the two beams go in opposite directions through the sample cell containing the atomic vapour. The pump beam interacts with atoms that have velocity $v = (\omega - \omega_0)/k$ and excites many of them into the upper level, as shown in Fig. 8.3(b). This is referred to as hole burning. The hole burnt into the lower-level population by a beam of intensity I has a width

$$\Delta\omega_{\text{hole}} = \Gamma \left(1 + \frac{I}{I_{\text{sat}}}\right)^{1/2}, \quad (8.19)$$

equal to the power-broadened homogeneous width in eqn 7.88.

When the laser has a frequency far from resonance, $|\omega - \omega_0| \gg \Delta\omega_{\text{hole}}$, the pump and probe beams interact with different atoms so the pump beam does not affect the probe beam, as illustrated on the left- and

¹⁸This treatment of saturation is restricted to two-level atoms. Real systems with degeneracy are more difficult to treat since, under conditions with significant saturation of the absorption, the atoms are usually not uniformly distributed over the sub-levels (unless the light is unpolarized). Nevertheless, the expression $N_1(v) - g_1 N_2(v)/g_2$ is often used for the difference in population densities in a given velocity class.

¹⁹Normally, we have $I_{\text{probe}} \ll I_{\text{sat}}$ and $I_{\text{pump}} \gtrsim I_{\text{sat}}$.

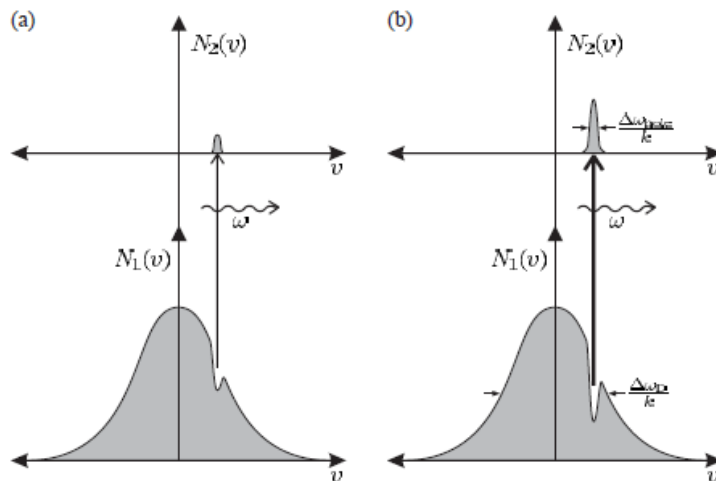


Fig. 8.3 The saturation of absorption. (a) A weak beam does not significantly alter the number density of atoms in each level. The number density in the lower level $N_1(v)$ has a Gaussian distribution of velocities characteristic of Doppler broadening of width $\Delta\omega_D/k$. The upper level has a negligible population, $N_2(v) \simeq 0$. (b) A high-intensity laser beam burns a deep hole—the population difference $N_1(v) - N_2(v)$ tends to zero for the atoms that interact most strongly with the light (those with velocity $v = (\omega - \omega_0)/k$). Note that $N_1(v)$ does not tend to zero: strong pumping of a two-level system never gives population inversion. This figure also shows clearly that Doppler broadening is inhomogeneous so that atoms interact in different ways within the radiation.

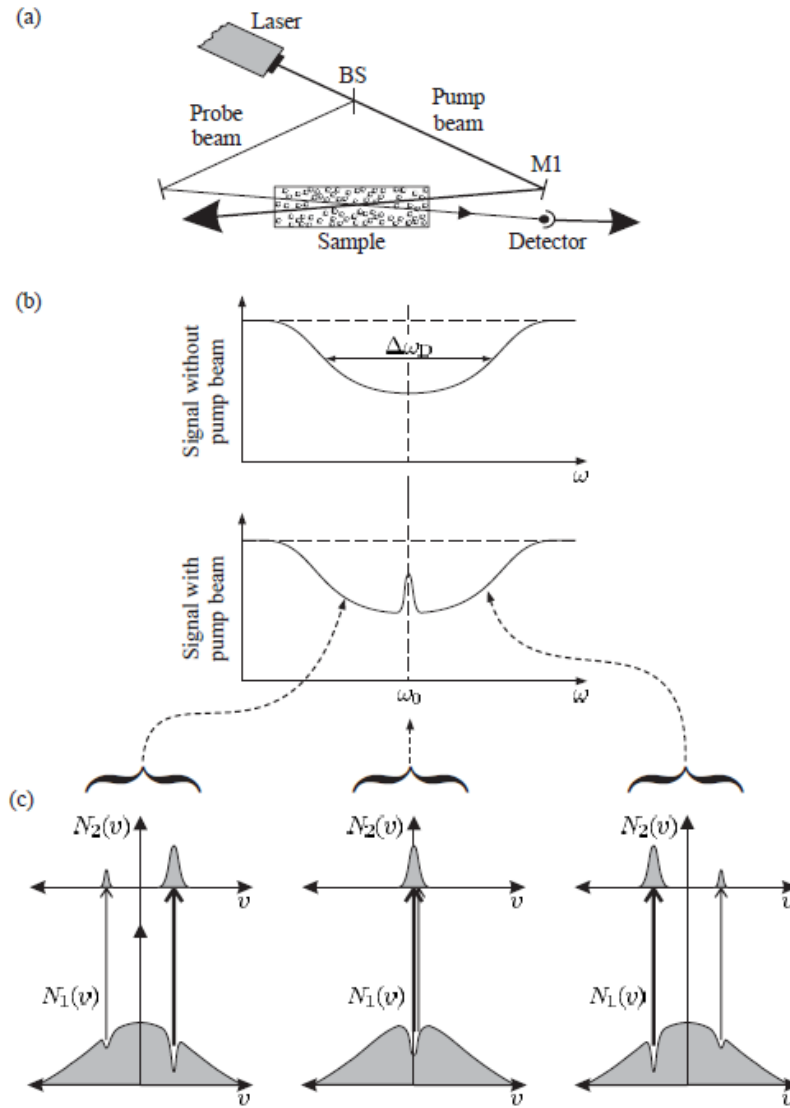


Fig. 8.4 (a) A saturated absorption spectroscopy experiment. The beam splitter BS, e.g. a piece of glass, divides the laser power between a weak probe and a stronger pump beam. The figure shows a finite angle of intersection between the weak probe beam and the stronger pump beam in the sample; this arrangement makes it straightforward to detect the probe beam after the cell but it leaves some residual Doppler broadening. Therefore saturated absorption experiments often have the pump and probe beams exactly counter-propagating and use a partially-reflecting mirror at M1 to transmit some of the probe beam to the detector (while still reflecting enough of the pump beam). (b) A plot of the probe intensity transmitted through the sample as a function of the laser frequency. With the pump beam blocked the experiment gives a simple Doppler-broadened absorption, but in the presence of the pump beam a narrow peak appears at the atomic resonance frequency. (c) The population densities of the two levels $N_1(v)$ and $N_2(v)$ as a function of velocity for three different laser frequencies: below, equal to, and above the atomic resonance, showing the effect of the pump and probe beams.

right-hand sides of Fig. 8.4(c). Close to resonance, $\omega \simeq \omega_0$, both beams interact with atoms in the velocity class with $v \simeq 0$, and the hole burnt by the pump beam reduces the absorption of the probe beam. Thus saturation of the absorption by the pump beam leads to a narrow peak in the intensity of the probe beam transmitted through the sample, as shown in Fig. 8.4(b). Normally, the pump beam has an intensity of about the saturation intensity I_{sat} , so the saturated absorption peaks always have a line width greater than the natural width. The *velocity class* of atoms that interact with the light has a velocity spread $\Delta v = \Delta\omega_{\text{hole}}/k$.

This section shows how saturation spectroscopy picks out a signal from the atoms in the velocity class centred at $v = 0$ to give a signal at the atomic resonance frequency. It is the homogeneous broadening of these stationary atoms that determines the widths of the peaks. Exercise 8.8 goes through a detailed calculation of this width. Many experiments use this Doppler-free technique to give a stable reference, e.g. to set the laser frequency a few line widths below resonance in laser cooling experiments with the optical molasses technique (described in the next chapter).²⁰

²⁰Nowadays, inexpensive semiconductor diode lasers make saturation spectroscopy a feasible experiment in undergraduate teaching laboratories, using the alkali elements rubidium or caesium that have sufficient vapour pressure at room temperature that a simple glass cell can be used as the sample (Wieman *et al.* 1999).

8.3.2 Cross-over resonances in saturation spectroscopy

In a saturated absorption spectrum, peaks appear at frequencies midway between pairs of transitions that have energy levels in common (and a separation less than the Doppler width), e.g. for the three-level atom shown in Fig. 8.5(a). To explain these *cross-over resonances* we need to consider the situation shown in Fig. 8.5(b), where the pump beam burns two holes in the velocity distribution. These holes give rise to two peaks in the spectrum when the laser frequency corresponds to the frequencies of the two transitions—the ‘expected’ saturated absorption signals for these two transitions. However, an additional peak appears when the hole burnt by one transition reduces the absorption for the other transition. As illustrated by Fig. 8.5(b), the symmetry of this situation means that cross-overs occur exactly midway between two saturated absorption peaks. This property allows experimenters to identify the cross-overs in a saturated absorption spectrum (see the exercises at the end of this chapter), and these extra peaks do not generally cause confusion.

The spectral lines of atomic hydrogen have large Doppler widths because it is the lightest element, but physicists want to measure the energy levels of this simple atom precisely to test atomic physics theory and to determine the Rydberg constant.

Figure 8.6 shows a spectrum of the Balmer- α line ($n = 2$ to $n = 3$) that is limited by Doppler broadening. This red line of atomic hydrogen, at a wavelength of $\lambda = 656$ nm, has a Doppler width of $\Delta f_D = 6$ GHz at room temperature (Section 8.1); this is less than the 11 GHz interval between the $j = 1/2$ and $3/2$ fine-structure levels in the $n = 2$ shell. Using the isotope deuterium (which has twice the atomic mass of hydrogen) in a discharge cooled to 100 K reduces the Doppler width to $\Delta f_D = 2.3$ GHz,²¹ where one factor arises from the mass and the other from the

²¹As calculated by scaling the value for hydrogen. The ratio of the Doppler widths for H at $T = 300$ K and D at $T = 100$ K is $\sqrt{6} = \sqrt{2} \times \sqrt{3}$.

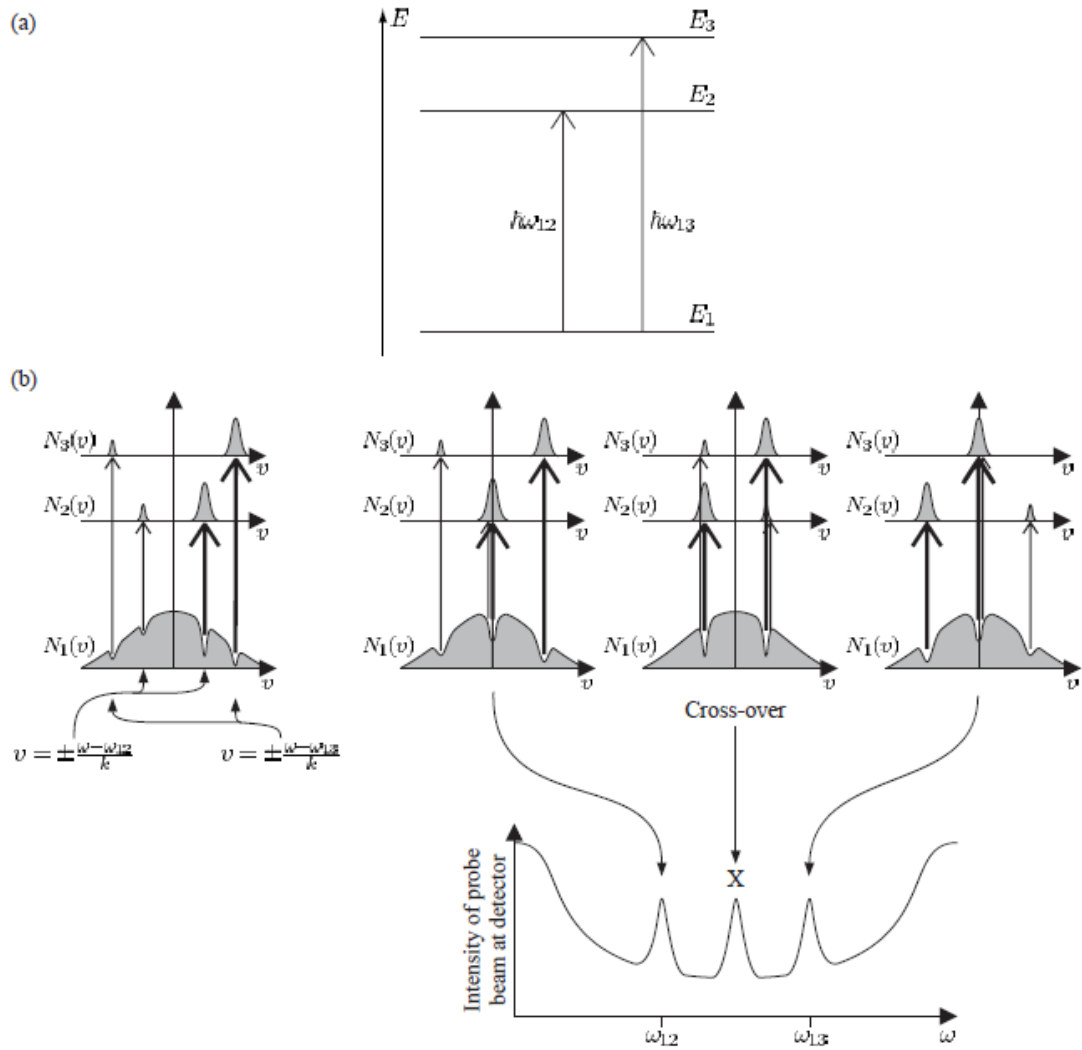


Fig. 8.5 The formation of a cross-over resonance. (a) A three-level atom with two allowed transitions at angular frequencies ω_{12} and ω_{13} . (b) A cross-over resonance occurs at X , midway between two saturated absorption peaks corresponding to transitions at angular frequencies ω_{12} and ω_{13} . At the cross-over the hole burnt by the pump beam acting on transition $1 \leftrightarrow 2$ reduces the probe beam absorption on transition $1 \leftrightarrow 3$, and vice versa.

²²The expectation value of the spin-orbit interaction scales as $1/n^3$ (in eqn 2.56) and hence the splitting for $n = 3$ is $8/27 \simeq 0.3$ times that for $n = 2$.

temperature. This makes it possible to observe components separated by the 3.3 GHz interval between the $j = 1/2$ and $3/2$ fine-structure levels in the $n = 3$ shell—see Figs 8.6(c) and 8.7(a).²² Structure on the scale of the 1 GHz corresponding to the Lamb shift cannot be resolved by conventional Doppler-limited techniques.

Figure 8.7 shows the spectacular improvement in resolution obtained with Doppler-free spectroscopy. The saturated absorption spectrum shown in Fig. 8.7(c) was obtained in a room-temperature discharge of

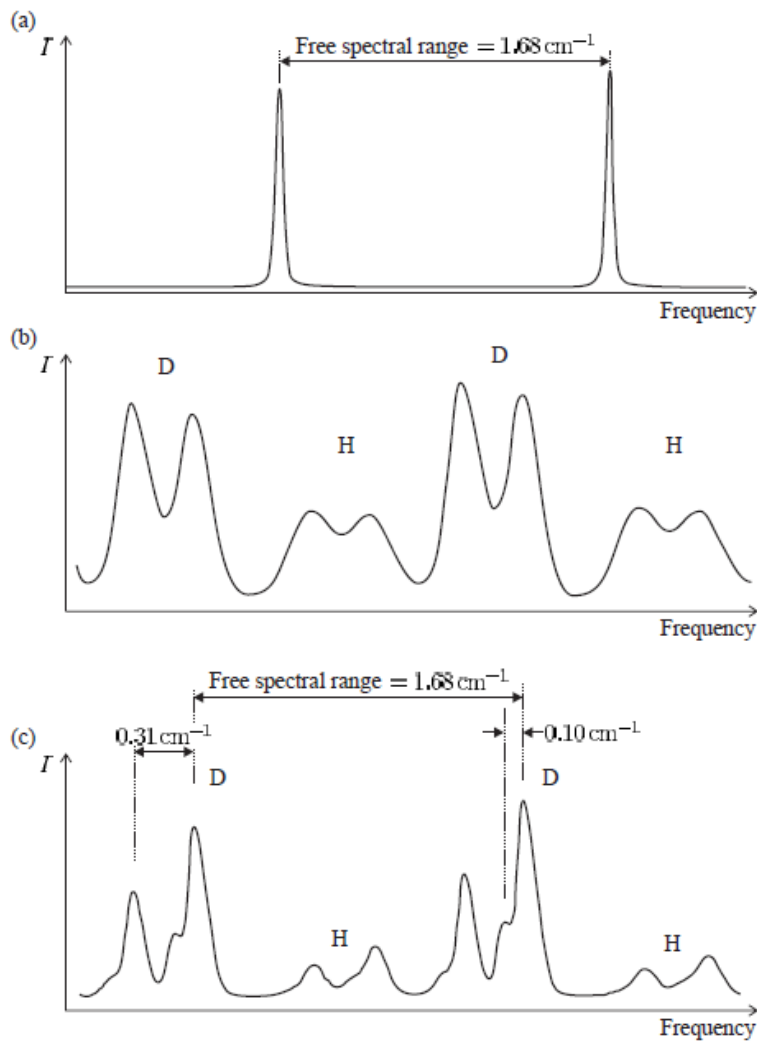
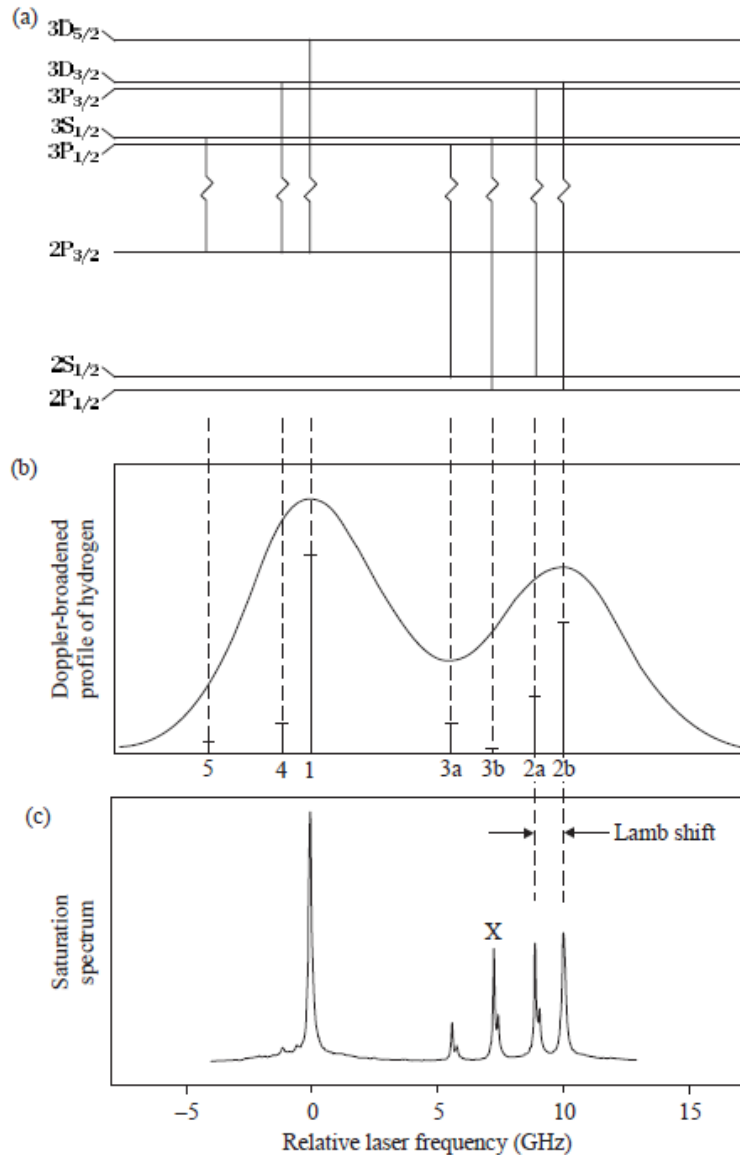


Fig. 8.6 Spectroscopy of the Balmer- α line, carried out with an apparatus similar to that in Fig. 1.7(a), has a resolution limited by the Doppler effect. (a) The transmission peaks of a pressure-scanned Fabry-Perot étalon obtained with a highly-monochromatic source (helium-neon laser). The spacing of the peaks equals the free-spectral range of the étalon given by $\text{FSR} = 1/2l = 1.68 \text{ cm}^{-1}$, where l is the distance between the two highly-reflecting mirrors. The ratio of the FSR to the width of the peaks (FWHM) equals the finesse of the étalon, which is about 40 in this case. (The difference in height of the two peaks in this trace of real data arises from changes in laser intensity over time.) In all the traces, (a) to (c), the étalon was scanned over two free-spectral ranges. (b) The spectrum from a discharge of hydrogen, H, and deuterium, D, at room temperature. For each isotope, the two components have a separation approximately equal to the interval between the fine-structure levels with $n = 2$. This splitting is slightly larger than the Doppler width for hydrogen. The isotope shift between the hydrogen and deuterium lines is about 2.5 times larger than the free-spectral range, so that adjacent peaks for H and D come from different orders of the étalon. (The étalon length has been carefully chosen to avoid overlap whilst giving high resolution.) (c) The spectrum of hydrogen and deuterium cooled to around 100 K by immersing the discharge tube in liquid nitrogen. (The relative intensities change with discharge conditions.) The fine structure of the 3p configuration is not quite resolved, even for deuterium, but leads to observable shoulders on the left of each peak—the relevant energy levels are shown in Fig. 8.7. Courtesy of Dr John H. Sanders, Physics department, University of Oxford.

atomic hydrogen and part of the spectrum in Fig. 8.6(b) is shown for comparison.²³ The saturated absorption technique gives clearly resolved peaks from the 2a and 2b transitions with a separation equal to the Lamb shift—the QED contributions shift the energy of the $2s \ ^2S_{1/2}$ level upwards relative to $2p \ ^2P_{1/2}$. Lamb and Retherford had measured this shift by a radio-frequency method using a metastable beam of hydrogen

²³The first saturated absorption spectrum of hydrogen was obtained by Professor Theodor Hänsch and co-workers at Stanford University (around 1972). In those pioneering experiments the width of the observed peaks was limited by the bandwidth of the pulsed lasers used. Continuous-wave lasers have lower bandwidth.

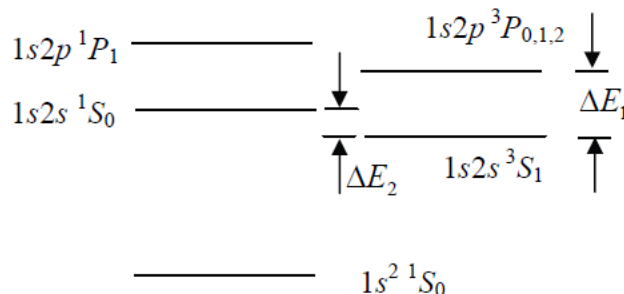
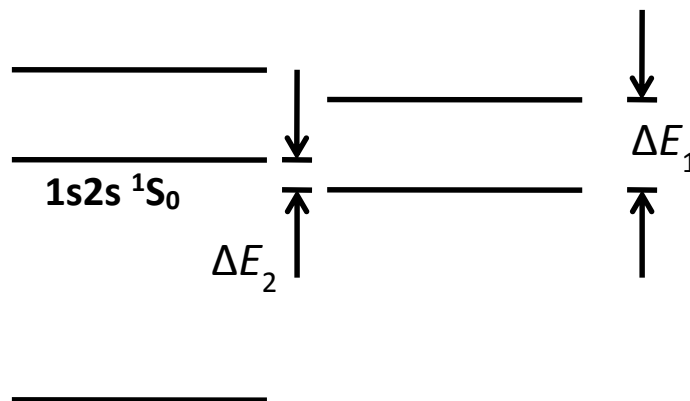
Fig. 8.7 Spectroscopy of the Balmer- α transition. (a) The levels with principal quantum numbers $n = 2$ and $n = 3$ and the transitions between them. Relativistic quantum mechanics (the Dirac equation) predicts that energies depend only on n and j , leading to the five transitions labelled 1 to 5 in order of decreasing strength (proportional to the square of the matrix element). In reality, some of these levels are not degenerate because of QED effects, e.g. the Lamb shift between $2s\ ^2S_{1/2}$ and $2p\ ^2P_{1/2}$ that gives two components in transitions 2 and 3. Thus there are seven optical transitions (that were listed in Section 2.3.5). (The allowed transition between the $2s\ ^2S_{1/2}$ and $2p\ ^2P_{1/2}$ levels, and other radio-frequency transitions are not marked.) (b) The Doppler-broadened profile of the Balmer- α line in a room-temperature discharge containing atomic hydrogen shows only two clear components separated by about 10 GHz (slightly less than the fine-structure splitting of the $2p$ configuration—see the caption of Fig. 8.6). (c) The saturated absorption spectrum obtained with a continuous-wave laser. The Lamb shift between the $2a$ and $2b$ components is clearly resolved. The $2s\ ^2S_{1/2}$ level has a hyperfine splitting of 178 MHz and this leads to the double-peaked profile of $2a$, $3a$ and the cross-over resonance X midway between them. (In addition to their relative positions, further evidence that peak X is the cross-over resonance between $2a$ and $3a$ comes from their similar line shape, which strongly indicates that they share a common level; the weak transition $3b$ is obscured.) Transition 4 is also seen on the far left and the cross-over resonance between 4 and 1 is just visible as a small bump on the base of peak 1. The scale gives the laser frequency relative to an arbitrary point (transition 1). Data shown in (c) was obtained by Dr John R. Brandenberger and the author.



Aufgabe 2: Das Helium-Atom (3P, typische Klausuraufgabe)

Die Skizze zeigt die fünf niedrigsten elektronischen Zustände des Heliumatoms.

- Tragen Sie entsprechend wie beim bezeichneten Niveau die Konfiguration der anderen vier Niveaus ein.
- Erklären Sie die Ursachen der Energiedifferenzen ΔE_1 und ΔE_2 . Wie entsteht der Energieunterschied zwischen Singulett und Triplet Zuständen?
- Warum gibt es keine optischen Übergänge zwischen Singulett und Triplet Zuständen?



Solution:

- see the labels in the figure.
- ΔE_1 comes from the difference of electronic configurations $1s2s$ and $1s2p$, where the $2s$ electron has a centrifugal potential of 0 and the $2p$ electron a centrifugal potential of $\frac{l(l+1)\hbar^2}{r}$;

ΔE_2 comes from the energy difference between singlet and triplet states, where the exchange energy makes the energy of the triplet state lower.

- the optically allowed transitions are: $1s2p\ ^1P_1 \rightarrow 1s2s\ ^1S_0$; $1s2p\ ^1P_1 \rightarrow 1s^2\ ^1S_0$; $1s2p\ ^3P_{0,1,2} \rightarrow 1s2s\ ^3S_1$

Aufgabe 4: *LS*- und *jj*-Kopplung (3P)

Nehmen Sie an, dass ein Zweielektronensystem $2p3s$ gut durch *LS*-Kopplung beschrieben sei, während ein System aus $6p7s$ gut durch *jj*-Kopplung beschrieben wird. Zählen Sie die möglichen Zustände für beide Systeme auf. Wie viele Zustände gibt es jeweils?

Solution:

$$s_1 = 1/2, s_2 = 1/2, l_1 = 1, l_2 = 0;$$

For *LS* coupling, one can get $S=0,1; L=1$. So the terms are $^1P_1, ^3P_{0,1,2}$. There are 4 terms.

For *jj* coupling, we have $j_1=l_1+s_1$, so $j_1=1/2, 3/2$; similarly, $j_2=1/2$. With $J=j_1+j_2$, one gets $J=0, 1; 1,2$. There are also 4 terms.