7)

$$|x|^2 = x \cdot x = \eta_{\mu\nu} x^\mu x^\nu$$

$$x'^\rho = \Lambda^\rho{}_\mu x^\mu$$

a) $|x'|^2 = |x|^2$

$$\eta_{\mu\nu} x^\mu x^\nu = \eta_{\rho\sigma} x'^\rho x'^\sigma = \eta_{\rho\sigma} \Lambda^\rho{}_\lambda x^\lambda \Lambda^\sigma{}_\omega x^\omega$$

$=$

$\eta_{\lambda\omega}$

5)

$$\Lambda^\nu_\mu(\psi) = \begin{pmatrix} \cosh\psi & -\sinh\psi & 0 & 0 \\ -\sinh\psi & \cosh\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} M(\psi) & 0 \\ 0 & 1 \end{pmatrix}$$

$$M(\psi)M(\bar\phi) = \begin{pmatrix} c\psi & -s\psi \\ -s\psi & c\psi \end{pmatrix}\begin{pmatrix} c\bar\phi & -s\bar\phi \\ -s\bar\phi & c\bar\phi \end{pmatrix}$$

$$= \begin{pmatrix} c\psi c\bar\phi + s\psi s\bar\phi & -c\psi s\bar\phi - s\psi c\bar\phi \\ -s\psi c\bar\phi - c\psi s\bar\phi & s\psi s\bar\phi + c\psi c\bar\phi \end{pmatrix}$$

$$= \begin{pmatrix} c(\psi+\bar\phi) & -s(\psi+\bar\phi) \\ -s(\psi+\bar\phi) & c(\psi+\bar\phi) \end{pmatrix} = \Lambda^\nu_\mu(\psi-\bar\phi)$$

- $\Lambda^\nu_\mu(\psi)\,\Lambda^\nu_\mu(\bar\phi) = \begin{pmatrix} M(\psi)M(\bar\phi) & 0 \\ 0 & 1 \end{pmatrix}$

- $\left[\Lambda^\nu_\mu(\psi)\,\Lambda^\nu_\mu(\bar\phi)\right]\Lambda^\nu_\mu(\Omega) = \begin{pmatrix} M(\psi)M(\bar\phi) & 0 \\ 0 & 1 \end{pmatrix}\Lambda^\nu_\mu(\Omega)$

$$= \begin{pmatrix} M(\psi+\bar\phi) & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} M(\Omega) & 0 \\ 0 & 1 \end{pmatrix} = M[(\psi+\bar\phi)+\Omega] = M[\psi+(\bar\phi+\Omega)]$$

$$= \Lambda^\nu_\mu(\psi)\left[\Lambda^\nu_\mu(\bar\phi)\,\Lambda^\nu_\mu(\Omega)\right]$$

- $e = \Lambda^\nu_\mu(0) = 1$ trivial

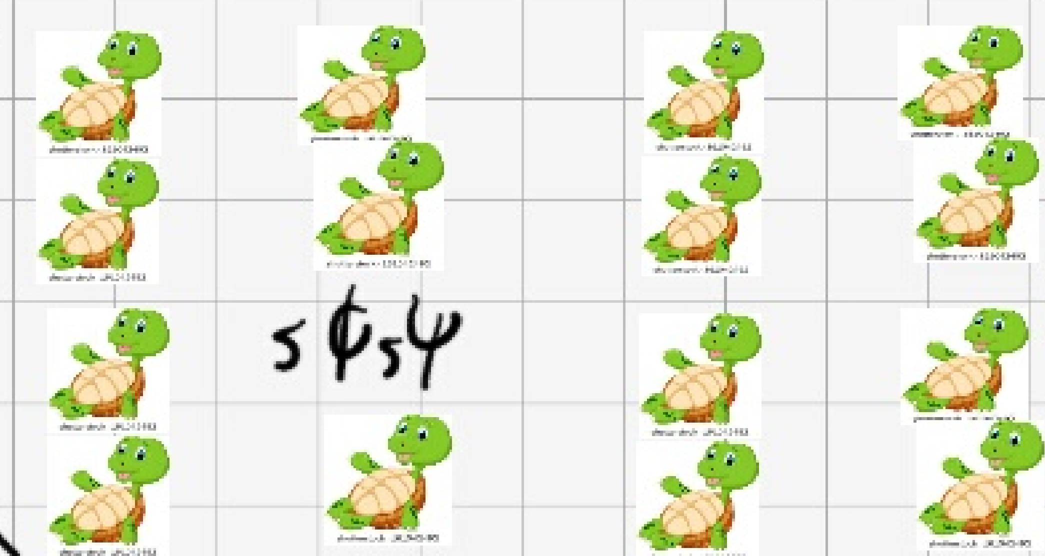
- guess: $\left(\Lambda^{\nu}{}_{\mu}(\psi)\right)^{-1} = \Lambda^{\nu}{}_{\mu}(-\psi)$

$$\Lambda^{\nu}{}_{\mu}(\psi)\left(\Lambda^{\nu}{}_{\mu}(\psi)\right)^{-1} = \Lambda^{\nu}{}_{\mu}(\psi)\,\Lambda^{\nu}{}_{\mu}(-\psi) = \Lambda^{\nu}{}_{\mu}(\psi-\psi) = \Lambda^{\nu}{}_{\mu}(0) = e$$

c) $\Lambda^{\nu}{}_{\mu}(\psi)\,\Lambda^{\nu}{}_{\mu}(\Phi) = \Lambda^{\nu}{}_{\mu}(\psi+\Phi) = \Lambda^{\nu}{}_{\mu}(\Phi+\psi) = \Lambda^{\nu}{}_{\mu}(\Phi)\,\Lambda^{\nu}{}_{\mu}(\psi)$ $\Rightarrow$ one way abelian

$$\begin{pmatrix} c\psi & -s\psi & 0 & 0 \\ -s\psi & c\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} c\phi & 0 & -s\phi & 0 \\ 0 & 1 & 0 & 0 \\ -s\phi & 0 & c\phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} c\psi c\phi & -s\psi & -c\psi s\phi & 0 \\ -s\psi c\phi & c\psi & s\psi s\phi & 0 \\ -s\phi & 0 & c\phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

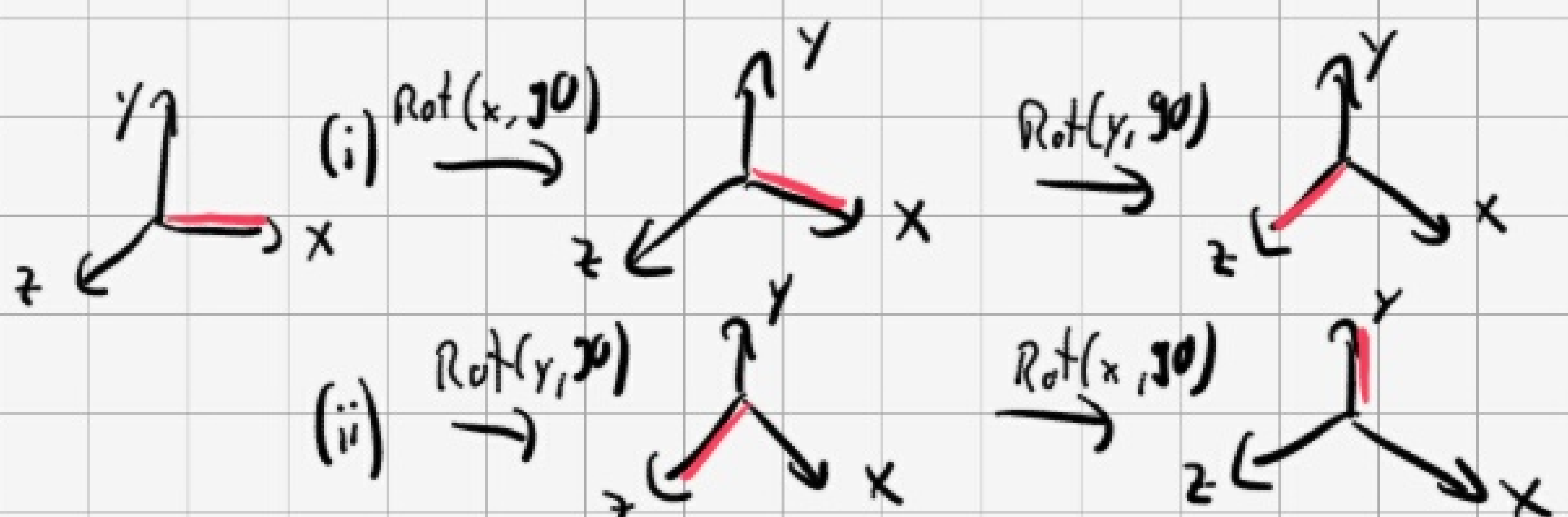
$$\begin{pmatrix} c\phi & 0 & -s\phi & 0 \\ 0 & 1 & 0 & 0 \\ -s\phi & 0 & c\phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} c\psi & -s\psi & 0 & 0 \\ -s\psi & c\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} & & & \\ & & & \\ & s\phi s\psi & & \\ & & & \end{pmatrix}$$



not abelian

extra

hammer



$\neq$ $\rightarrow$ not abelian

d)

$$A \Rightarrow \exp(A) = \sum_n \frac{A^n}{n!}$$

$\cos(\psi L)$ $\sin(\psi L)$

$$\Lambda^\nu_\mu(\psi) = \sum_n \frac{(\psi L)^n}{n!} = \sum_n \frac{(\psi L)^{2n}}{2n!} + i\sum_n \frac{(\psi L)^{2n+1}}{(2n+1)!}$$

$$e^x = \sum_n \frac{x!}{n!}$$

$$\Lambda^\nu_\mu = \begin{pmatrix} \cos(ix) & i\sin(ix) & 0 & \\ i\sin(ix) & \cos(ix) & & \\ & 0 & 1 & \\ & & & 1 \end{pmatrix}$$

$$e^x = \cos(x) + i\sin(x)$$