Zeeman Spectroscopy [1]

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Abstract: In this experiment, we successfully show the Zeeman effect in a cadmium lamp. Firstly, we use a Lummer-Gehrcke-Plate and a telescope to qualitatively observe the splitting of different spectral lines. We then make a quantitative measurement using a CCD-camera. We use a separate high performance spectrometer to characterise the spectrum of the lamp and determine the wavelength of the lines.

Attested as special evaluation: Date, Signature:

[1] Experiment F44, performed 27.01.15, Supervisor: Mykola Stepaniuk, long evaluation

1 Introduction

The Zeeman effect, which was discovered by dutch physicist Pieter Zeeman, describes the splitting of the spectral lines of an atom placed in a static magnetic field. This splitting occurs due to different magnetic moments of the respective orbitals. By this, a variety of different applications in atomic and molecular physics is made possible. To detect and measure this effect the very important experimental method of spectroscopy is used. The fundamental principle used in spectroscopy is that light of different energies has different wavelengths and frequencies.

$$E = h\nu \tag{1}$$

$$c = \nu\lambda \tag{2}$$

This results in different behaviour upon interaction with matter. Thus one can use a certain set of spectroscopic methods to identify different energies of a given light source.

The atomic structure of atoms is determined by the energy differences of the respective levels. When electrons are excited, they occupy higher energy levels. Upon falling back to the original level through spontaneous emission, they lose energy in the form of photons, whose energy and thus wavelength is determined by the energy distance between the levels. Therefore one can analyse the atomic structure with spectroscopic methods, which will be explained later on.

2 Theory

2.1 Normal Zeeman effect

The spin-less case is called *normal* Zeeman effect. Here, the total angular momentum of the electron comes from the orbital momentum. An external magnetic field couples to this angular magnetic moment. By that, it changes the electron's potential energy by

$$\Delta E_{pot} = -\vec{\mu}_l \cdot \mathbf{B} = \frac{e}{2m_e} \cdot \mathbf{l} \cdot \mathbf{B} \tag{3}$$

Using quantisation in z direction, this can be simplified to

$$\Delta E_{pot} = \frac{e \cdot \hbar}{2m_e} \cdot m_l \cdot B = \mu_B \cdot m_l \cdot B, \tag{4}$$

where μ_B is called the Bohr magneton. The energy shift leads to a splitting of the original level to $2l + 1$ sublevels of the same angular momentum l but different magnetic quantum numbers m_l. So the energy distance between two levels is

$$\Delta E = \mu_B \cdot B. \tag{5}$$

2.2 Selection rules and polarisation

Quantum mechanical calculations (the transition dipole matrix) require for transitions between Zeeman levels that the change in the magnetic quantum number fulfils $\Delta M_J = 0, \pm 1$. Furthermore, they require the total orbital momentum $\Delta L = \pm 1$ and constancy of spin, i. e. $\Delta S = 0$.

From calculating the transition dipole matrix, one can deduce that in the case of $\Delta M_J = 0$, the light is emitted by a dipole oriented in z direction, which means one can observe light that is linearly polarised along the z direction when viewing from the x or y direction, and no light when viewing from z direction. This transition is called a π-transition.

In the case of $\Delta M_J = \pm 1$, the light is emitted by two superimposed dipoles in x and y direction which are phase-shifted by $\frac{\pi}{2}$. This leads to the impression of right-handed ($\Delta M_J = +1$, so-called σ^+-transition) or left-handed ($\Delta M_J = -1$, so-called σ^--transition) circular polarised light when viewed in z direction. In x and y directions, one can only observe the projections of one of the dipoles and thus sees only linear polarised light, perpendicular to the z axis.

3 Experimental setup and techniques

The experiment consists of two major parts. We begin by observing the Zeeman effect in a cadmium lamp. We use the splitting of the different spectral lines to calculate the Bohr magneton. To achieve this, we expose the light source to a strong magnetic field and observe the spectrum. In the second part of the experiment we are using a Czerny-Turner spectrometer and a neon light source as reference to measure the wavelength of the used cadmium spectral line.

3.1 Observation of the Zeeman spectrum

To measure the Zeeman spectrum, we use a spectrometer whose main part is the so called Lummer-Gehrcke plate. It is a glass plate in which incoming light is totally reflected multiple times, while only a small portion leaves the plate at each reflection.

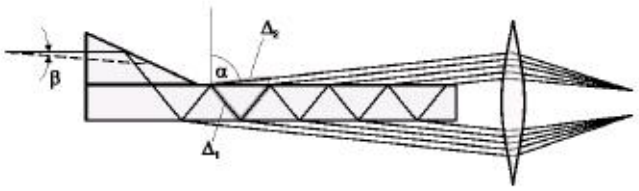

Figure 1: Scheme of the Lummer-Gercke interferometer. It consist of a long plate of glass whose faces are flat and very precisely parallel. Incoming light is reflected multiple times along the plate, while only a small portion of the beam leaves the glass at every bounce. A lens focusses all these beams, which gives the interference pattern.

All these beams then interfere above and under the plate and can be observed by the telescope. To observe the Zeeman effect it is necessary to expose the light source to a strong magnetic field, which is provided by a large pair of coils which are connected by iron rods. The whole setup can be seen in Figure 2.

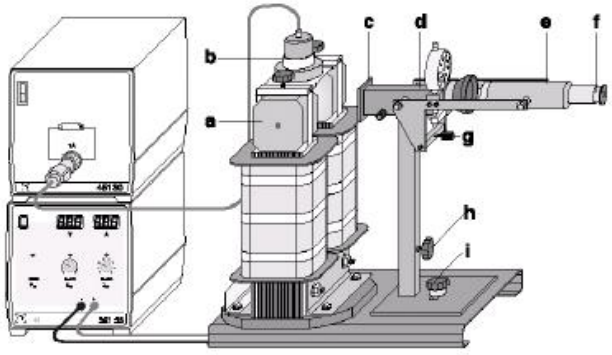

Figure 2: Setup for measuring the Zeeman effect. The light source is a cadmium lamp (b), that is exposed to different magnetic fields of a strong magnet (a). The observation is done using a telescope (e, f) and a Lummer-Gehrcke plate (c, d), that allows the measurement of the Bohr magneton.

An interesting effect is the so called hysteresis. The iron gets magnetised and therefore, theoretically, the magnetic field should be lower when measuring it while increasing the current than when decreasing it. For the experiment we measured the magnetic field both ways. We used a small Hall probe that we would hold at the position of the lamp. To decrease the error, we measured three times at each current. The results can be seen in Figure 3.

3.2 Czerny-Turner spectrometer

For finding the wavelength of the cadmium lamp we have to compare it with a known spectrum, which in our case is neon. To make high precision measurements of both spectra we use a so called Czerny-Turner spectrometer. Via optical fibres the light of the cadmium and neon lamp is send through a slit into the spectrometer (Figure 4).

To measure the spectrum, we use a CCD camera that is placed behind the exit slit. The camera we used has a resolution of 2048x512 pixels. To reduce thermal noise and increase signal to noise ratio, the device is cooled down with liquid nitrogen. The data is read with the SpectraMax software.

The images we get show intensities for each column of pixels. Therefore the scale is arbitrary and we have to calibrate the scale with use of the known spectrum of our neon lamp.

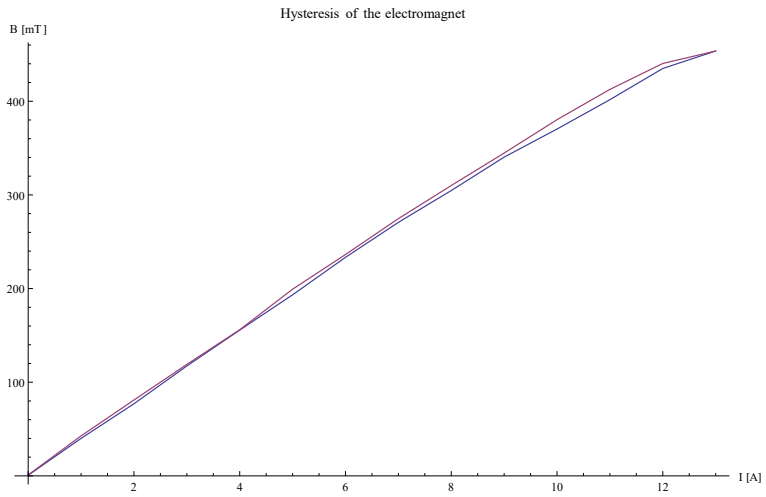

Figure 3: Hysteresis of the magnet. We measured the magnetic field while increasing (blue) and decreasing (pink) the current. For better visibility, the error bars are not included. The hysteresis is visible especially in the region of high current. Below 10 A there is no significant effect. Above this current the effect is significant but not strong. However it is important for the experiment, because the Zeeman splitting will be measured in high magnetic fields.

4 Measurement and Evaluation

4.1 Measurement of the Bohr magneton

4.1.1 Theory

The main goal of the experiment is to determine the Bohr magneton μ_B. This is achieved by determining the difference in wavelengths between the split lines. After adjusting the telescope, we can see spectral lines of different color. For the measurement we select the red spectral line of cadmium (643.8 nm, see section 4.2) with a red filter. With increasing magnetic field, the splitting of the lines becomes visible. We use a polarisation filter to check the polarisation features of the different lines. In transversal direction, the different lines can be suppressed.

μ_B can be determined from the energy difference between the split lines. The energy difference is proportional to a change in wavelength, which in case of our Lummer-Gehrcke plate is given by [2]:

$$\Delta\lambda = \frac{\delta a}{\Delta a}\frac{\lambda^2}{2d\sqrt{n^2-1}} \tag{6}$$

$$\Delta E = hc\left(\frac{1}{\lambda-\Delta\lambda} - \frac{1}{\lambda}\right) \tag{7}$$

The values d (thickness) and n (refractive index) are fixed properties of the Lummer-Gehrcke plate and λ is measured in the next part of the experiment. The remaining values are δa and Δa,

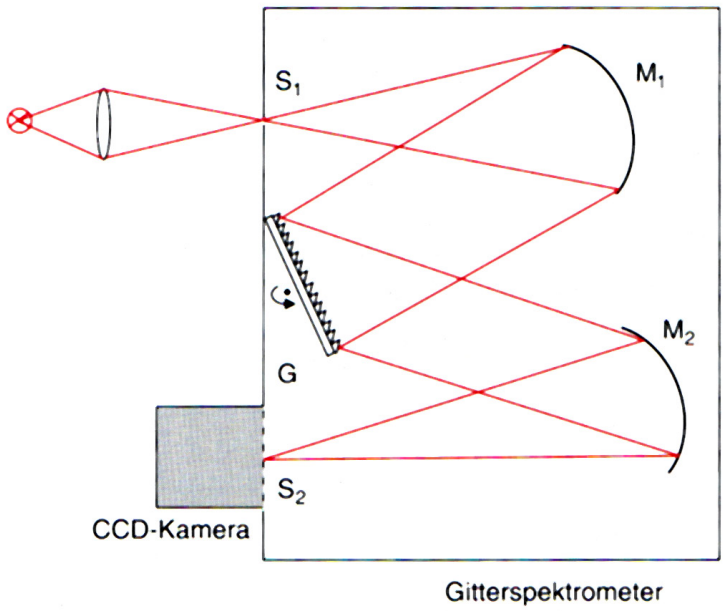

Figure 4: Scheme of a Czerny-Turner spectrometer. The light enters via a small entrance slit. A curved mirror collimates the light and reflects it onto a reflective grating. The beam then is projected onto another mirror, whose focus lies on the exit slit. Because of the dispersion of the grating the exiting light is nearly monochromatic. By turning it, one can select a certain wavelength and scanning over a certain spectrum. Behind the slit, a CCD-camera measures the intensity of each wavelength.

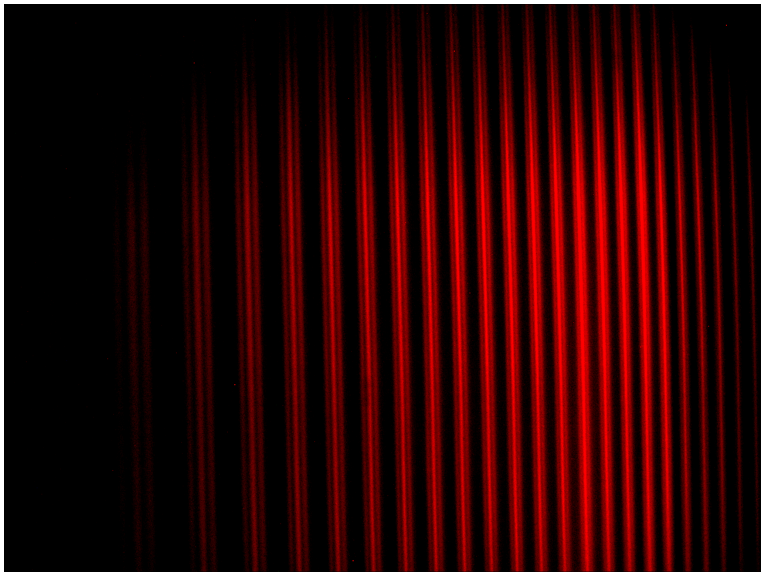

Figure 5: Picture of the CCD-camera, through the telescope. The different triplets of lines refer to higher orders of interference in the Lummer-Gehrcke plate. To measure the strength of the splitting the distances between the orders and between the maxima of one order are compared.

which are the distances between the π and σ lines and between the π lines of the neighbouring orders of interference, respectively.

4.1.2 Measurement

To determine these numbers, we analyse the images made by our CCD-camera. Using the program ImageJ, we can select each line feature and average the intensity over the vertical direction. The resulting file gives us an intensity for each pixel of the camera. Because are only interested in relative distances, not in the absolute values of δa and Δa, this is sufficient without a gauge for the distance scale. The positions on the arbitrary scale are determined in Origin. The different line features are selected and fitted with help of the respective tool in the program.

4.1.3 Determination of the line positions

A Voigt profile is selected as fit function. This is explained as follows. A natural line shape is described by a Lorentz function. At operating temperature, however, the Doppler broadening has a significant influence, because the light of atoms moving towards and away from the observer is shifted to the blue and the red. The shape of the respective distribution, due to Maxwell-Boltzman statistics, is a Gaussian profile. The Voigt profile is a convolution of both. The comparison of all functions fitted to our points can be seen in Figure 7. To judge the quality of each profile, we compared the values of χ^2_{red}. The closer its value is to 1, the better the function fits the points of data. This shows that the Voigt function gives the best fit quality, followed by the Gaussian. The least successful result is given by the Lorentz-function, due to high temperature Doppler-broadening.

After finding the positions of all lines, we determine $\delta a/\Delta a$, which then is used to find $\Delta\lambda$ and ΔE.

4.1.4 Calculation of μ_B

The conceptually easiest method to calculate $\delta a/\Delta a$ is by directly using the pixel positions of the lines and use the relations

$$\frac{\delta a}{\Delta a} = \frac{a_{\pi,\,n} - a_{\sigma^+,\,n}}{a_{\pi,\,n} - a_{\pi,\,n+1}} \tag{8}$$

$$\frac{\delta a}{\Delta a} = \frac{a_{\pi,\,n} - a_{\sigma^-,\,n}}{a_{\pi,\,n} - a_{\pi,\,n-1}}, \tag{9}$$

where the index n stands for the order of interference.

We use all measured data and average over all $\delta a/\Delta a$ values for a given magnetic field strength to obtain one value with error (standard deviation).

A somewhat more sophisticated method works as follows: We plot the order of interference against the central positions of the π lines. To get a continuous fit of the discrete number

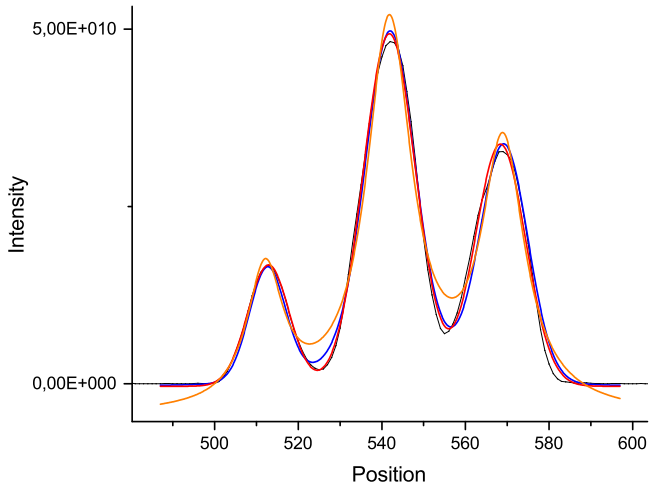

Figure 6: Fit-functions compared. Voigt profile (red), Gaussian profile (blue), Lorentz function (orange). The quality difference between the Lorentz function and both Gaussian and Voigt profile can easily be seen. The comparison between the last two can be done using χ^2_{red}. In our case, the Voigt function yields the best result.

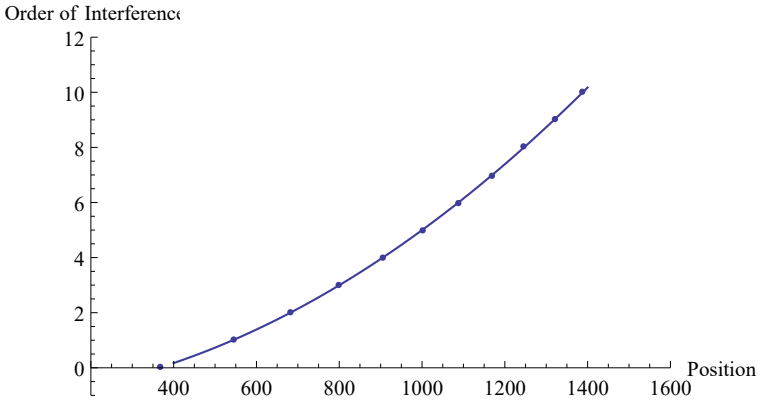

Figure 7: Order of interference against the position of the π lines.

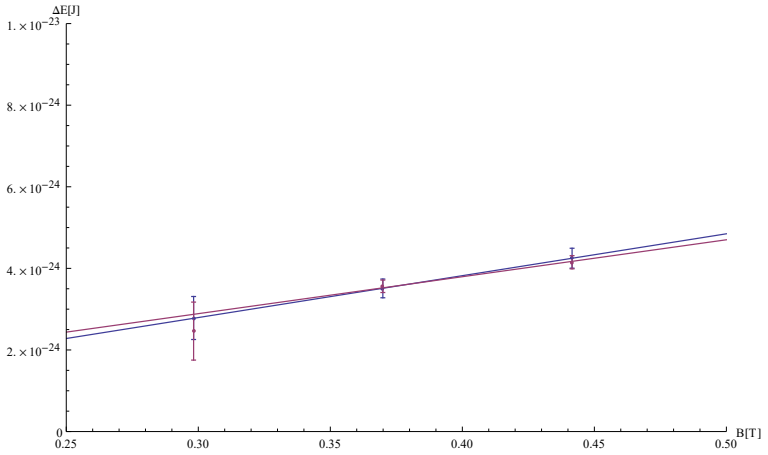

Figure 8: Calculating μ_B as slope of ΔE.

of orders, we fit a polynomial function to the points. The order of 2 for the polynomial is sufficient. We get a resulting function with the following parameters:

$$k_{8A}(a) = -1.12942 + 0.00114a + 4.87 \cdot 10^{-6} \tag{10}$$
$$k_{10A}(a) = -1.09804 + 0.00117a + 4.88349 \cdot 10^{-6} \tag{11}$$
$$k_{12A}(a) = -1.08333 + 0.00118a + 4.90012 \cdot 10^{-6} \tag{12}$$

Using the positions of the σ^+ lines, we determine $\delta a/\Delta a$ by the following formula:

$$\frac{\delta a}{\Delta a} = k(a_{\sigma^+}) - \lfloor k(a_{\sigma^+}) \rfloor \tag{13}$$

We calculate $\delta a/\Delta a$ for each order of interference to get mean values and standard errors.

Now we can use equation 6 to determine $\Delta\lambda$. Using $E = hc/\lambda$, this can be converted to ΔE. Using equation 5 we can calculate μ_B in two ways. The first way is to simply calculate the value for the three magnetic fields and average the results. Since μ_B is the slope of ΔE, we also can plot it against B and make a linear fit (Figure 8). The resulting slope parameter gives the value.

4.1.5 Results

We measured the following magnetic field strengths:

$$B_{8A} = 298.227 \text{ mT} \tag{14}$$
$$B_{10A} = 369.894 \text{ mT} \tag{15}$$
$$B_{12A} = 441.562 \text{ mT} \tag{16}$$

By just calculating the relations of the pixel positions, we obtain

Current	**B** [mT]	δ**a**/Δ**a**	Δ**E** [J]	$\mu_{\mathbf{B}}$ [J/T]
8 A	298.227	0.12 ± 0.02	$2.8 \pm 0.5 \cdot 10^{-24}$	$9.3 \pm 1.8 \cdot 10^{-24}$
10 A	369.894	0.15 ± 0.01	$3.5 \pm 0.2 \cdot 10^{-24}$	$9.5 \pm 0.6 \cdot 10^{-24}$
12 A	441.562	0.18 ± 0.01	$4.2 \pm 0.2 \cdot 10^{-24}$	$9.6 \pm 0.5 \cdot 10^{-24}$

Using the quadratic fit method and equation 13, we find

Current	**B** [mT]	δ**a**/Δ**a**	Δ**E** [J]	$\mu_{\mathbf{B}}$ [J/T]
8 A	298.227	0.106 ± 0.031	$2.4 \pm 0.7 \cdot 10^{-24}$	$8.3 \pm 2.4 \cdot 10^{-24}$
10 A	369.894	0.153 ± 0.007	$3.6 \pm 0.2 \cdot 10^{-24}$	$9.6 \pm 0.4 \cdot 10^{-24}$
12 A	441.562	0.178 ± 0.007	$4.2 \pm 0.2 \cdot 10^{-24}$	$9.4 \pm 0.4 \cdot 10^{-24}$

Plotting the ΔE values against B and fitting linearly (compare figure 8), we obtain

Method	**Slope** [J/T]
Pixel positions	$10.2 \pm 3.5 \cdot 10^{-24}$
Quadratic fit	$9.0 \pm 2.8 \cdot 10^{-24}$

4.2 Wavelength of the red cadmium spectral line

Using the Czerny-Turner spectrometer, we compare the spectrum of the cadmium lamp to the neon spectrum and find the wavelength of the red line, that we use for the determination of the Bohr magneton.

4.2.1 Calibration

To calibrate our scale we select a window of four neon lines. To find the positions of the lines we use a Voigt profile as fitting function. Like in the first part, we use a Voigt profile to fit our peaks. From the fit parameters we get the positions of the peaks. These can be compared to the literature value in table 1. We then use a linear fit to find a gauge for our scale. The resulting fit can be seen in figure 9.

$$\lambda = (657.97 - 0.01019x)\text{nm} \tag{17}$$

4.2.2 Wavelength measurement

Using the calibration we can determine the wavelength of the red cadmium line. Using the same scale and same method we find the positions of both lines in the vicinity of the neon references. One of them belongs to cadmium, the other is unknown. Using the NIST-Database [1], we are able to identify the element to be tungsten.

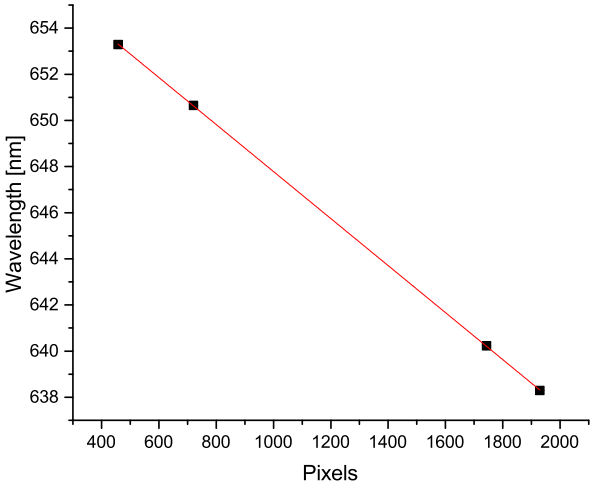

Figure 9: Calibration of the Czerny-Turner spectrometer. We compare literature values of the neon lines to their appearance on the arbitrary scale. A linear fit gives a gauge-function to find the wavelength of the cadmium line.

Pixels	**Lit. Wavelength [nm]**
457 ± 4	653.28824
721 ± 3	650.65277
1743 ± 5	640.2248
1929 ± 4	638.29514

Table 1: Neon lines for reference. Pixels (the errors are the FWHM) and their respective wavelength.

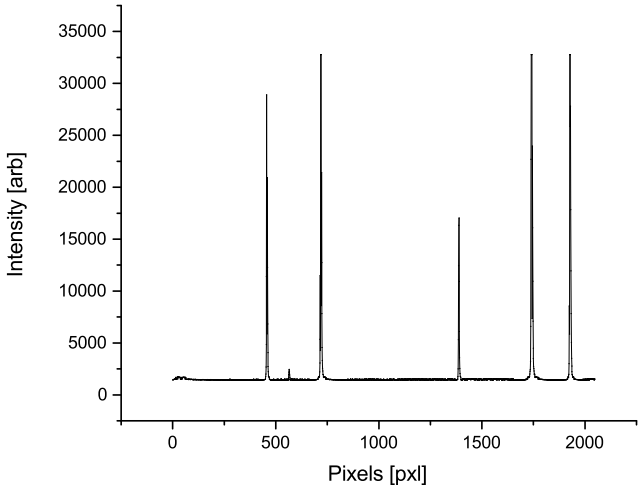

Figure 10: Plot of the spectra of both neon and cadmium. The two smaller lines belong to the cadmium lamp. The four big lines are the neon references.

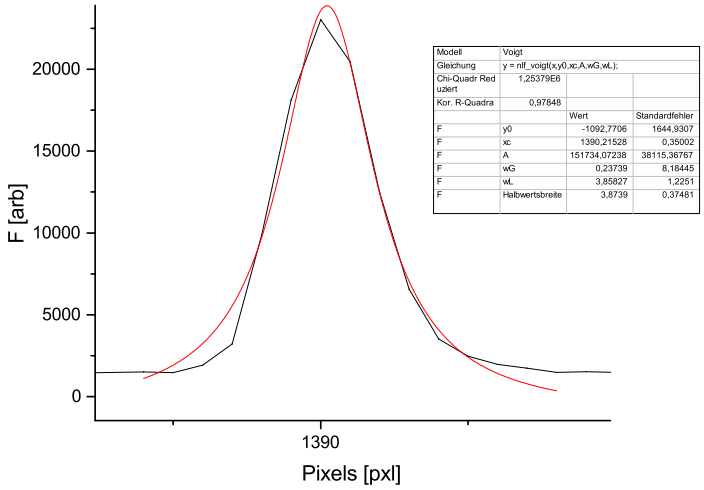

Figure 11: Fit of the red cadmium-line. A Voigt profile is fitted to determine the position of the spectral line The error is taken from the parameters.

Using equation 17 we can also determine an error for our value. The input value is the error of the Voigt fit.

Line	**Pixels**	**Wavelength [nm]**	**Lit. Wavelength [nm]**
Red Cadmium Line	1390 ± 4	643.80 ± 0.05	643.8470
Tungsten Line	567 ± 4	652.19 ± 0.04	652.198

Table 2: Measured values for both lines in the cadmium spectrum.

5 Discussion

In the first part of the experiment, we were successfully able to observe the Zeeman splitting in the spectrum of a cadmium lamp. We could identify the π and $\sigma^{\pm}$ lines using polarisation filters. Using the CCD camera, we could quantitatively observe the splitting and use these data to determine the Bohr magneton using several different methods of analysis. All methods are within their respective errors compliant to the literature value of $927.4009994(57) \cdot 10^{-26} \frac{\mathrm{J}}{\mathrm{T}}$ [3]. The method with the highest accuracy appears to be fitting the interference orders against the position of the π lines and determining the relationship between the splitting distance and the distance between the interference orders by interpolating the interference orders of the $\sigma^{\pm}$ lines. Because of the large errors present on the energy splitting values, the slope method turned out to be rather inaccurate in our setup.

The voltage generator used in our experiment had a maximum voltage, which we saturated. This led to the issue that we could not feed the electromagnet with higher currents. However, the higher the current, the stronger the magnetic field, and thus the splitting of the spectral lines. So with a higher current we could have observed stronger splitting, leading to higher precision measurement values.

In the second part of the experiment, we used a Czerny-Turner spectrometer to determine the wavelength of the Cadmium line. We were able to determine the wavelength of the cadmium line to 643.80 ± 0.05 nm, which is within one sigma distance of the literature value of 643.8470 nm.

We could identify the unknown present element as tungsten, where our measured wavelength of 652.19 ± 0.04 nm identifies with the literature value of 652.198 nm.

References

[1] Kramida, A., Ralchenko, Yu., Reader, J., and NIST ASD Team (2014). *NIST Atomic Spectra Database (ver. 5.2),* [Online]. Available: http://physics.nist.gov/asd [2015, August 13]. National Institute of Standards and Technology, Gaithersburg, MD.

[2] *Zeeman effect.*
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[3] Taylor, B. N., Mohr, P. J. et al. *The NIST Reference on Constants, Units, and Uncertainty,* [Online]. Avaliable; http://physics.nist.gov/cgi-bin/cuu/Value?mub [2015, August 19]. National Institute of Standards and Technology, Gaithersburg, MD.