CHAPTER 15

MARKET DEMAND

We have seen in earlier chapters how to model individual consumer choice. Here we see how to add up individual choices to get total **market demand**. Once we have derived the market demand curve, we will examine some of its properties, such as the relationship between demand and revenue.

15.1 From Individual to Market Demand

Let us use $x_i^1(p_1, p_2, m_i)$ to represent consumer i's demand function for good 1 and $x_i^2(p_1, p_2, m_i)$ for consumer i's demand function for good 2. Suppose that there are n consumers. Then the **market demand** for good 1, also called the **aggregate demand** for good 1, is the sum of these individual demands over all consumers:

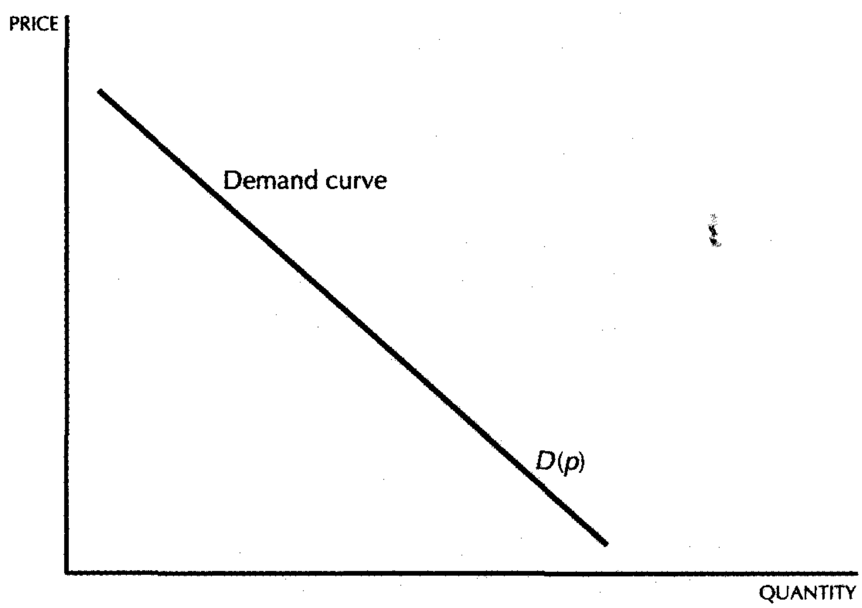
$$X^1(p_1, p_2, m_1, \ldots, m_n) = \sum_{i=1}^{n} x_i^1(p_1, p_2, m_i).$$

The analogous equation holds for good 2.

Since each individual's demand for each good depends on prices and his or her money income, the aggregate demand will generally depend on prices and the *distribution* of incomes. However, it is sometimes convenient to think of the aggregate demand as the demand of some "representative consumer" who has an income that is just the sum of all individual incomes. The conditions under which this can be done are rather restrictive, and a complete discussion of this issue is beyond the scope of this book.

If we do make the representative consumer assumption, the aggregate demand function will have the form $X^1(p_1, p_2, M)$, where M is the sum of the incomes of the individual consumers. Under this assumption, the aggregate demand in the economy is just like the demand of some individual who faces prices (p_1, p_2) and has income M.

If we fix all the money incomes and the price of good 2, we can illustrate the relation between the aggregate demand for good 1 and its price, as in Figure 15.1. Note that this curve is drawn holding all other prices and incomes fixed. If these other prices and incomes change, the aggregate demand curve will shift.



The market demand curve. The market demand curve is the sum of the individual demand curves.

For example, if goods 1 and 2 are substitutes, then we know that increasing the price of good 2 will tend to increase the demand for good 1 whatever its price. This means that increasing the price of good 2 will tend to shift the aggregate demand curve for good 1 outward. Similarly, if goods 1 and 2 are complements, increasing the price of good 2 will shift the aggregate demand curve for good 1 inward.

If good 1 is a normal good for an individual, then increasing that individual's money income, holding everything else fixed, would tend to increase that individual's demand, and therefore shift the aggregate demand curve outward. If we adopt the representative consumer model, and suppose that good 1 is a normal good for the representative consumer, then any economic change that increases aggregate income will increase the demand for good 1.

15.2 The Inverse Demand Function

We can look at the aggregate demand curve as giving us quantity as a function of price or as giving us price as a function of quantity. When we want to emphasize this latter view, we will sometimes refer to the **inverse demand function**, $P(X)$. This function measures what the market price for good 1 would have to be for X units of it to be demanded.

We've seen earlier that the price of a good measures the marginal rate of substitution (MRS) between it and all other goods; that is, the price of a good represents the marginal willingness to pay for an extra unit of the good by anyone who is demanding that good. If all consumers are facing the same prices for goods, then all consumers will have the same marginal rate of substitution at their optimal choices. Thus the inverse demand function, $P(X)$, measures the marginal rate of substitution, or the marginal willingness to pay, of *every* consumer who is purchasing the good.

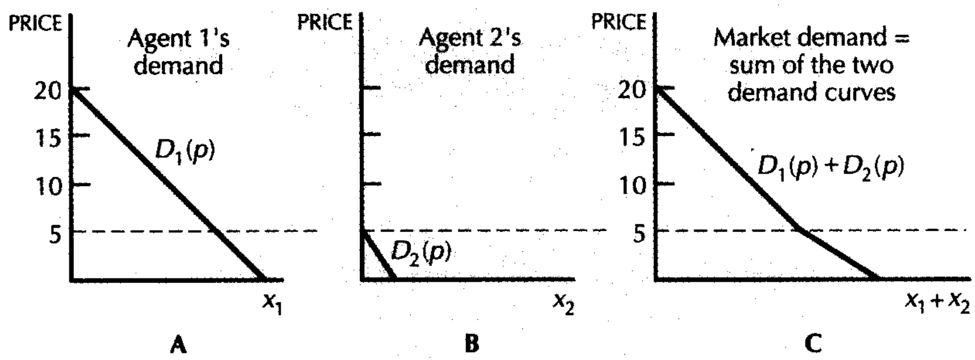
The geometric interpretation of this summing operation is pretty obvious. Note that we are summing the demand or supply curves *horizontally*: for any given price, we add up the individuals' quantities demanded, which, of course, are measured on the horizontal axis.

EXAMPLE: Adding Up "Linear" Demand Curves

Suppose that one individual's demand curve is $D_1(p) = 20 - p$ and another individual's is $D_2(p) = 10 - 2p$. What is the market demand function? We have to be a little careful here about what we mean by "linear" demand functions. Since a negative amount of a good usually has no meaning, we *really* mean that the individual demand functions have the form

$$D_1(p) = \max\{20 - p, 0\}$$
$$D_2(p) = \max\{10 - 2p, 0\}.$$

What economists call "linear" demand curves actually aren't linear functions! The sum of the two demand curves looks like the curve depicted in Figure 15.2. Note the kink at $p = 5$.



The sum of two "linear" demand curves. Since the demand curves are only linear for positive quantities, there will typically be a kink in the market demand curve.

15.3 Discrete Goods

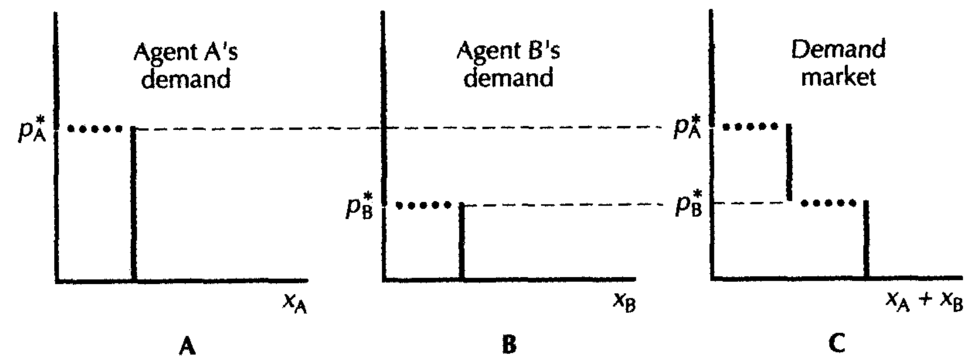
If a good is available only in discrete amounts, then we have seen that the demand for that good for a single consumer can be described in terms of the consumer's reservation prices. Here we examine the market demand for this kind of good. For simplicity, we will restrict ourselves to the case where the good will be available in units of zero or one.

In this case the demand of a consumer is completely described by his reservation price—the price at which he is just willing to purchase one unit. In Figure 15.3 we have depicted the demand curves for two consumers, A and B, and the market demand, which is the sum of these two demand curves. Note that the market demand curve in this case must "slope downward," since a decrease in the market price must increase the number of consumers who are willing to pay at least that price.

15.4 The Extensive and the Intensive Margin

In preceding chapters we have concentrated on consumer choice in which the consumer was consuming positive amounts of each good. When the price changes, the consumer decides to consume more or less of one good or the other, but still ends up consuming some of both goods. Economists sometimes say that this is an adjustment on the **intensive margin**.

In the reservation-price model, the consumers are deciding whether or not to enter the market for one of the goods. This is sometimes called an adjustment on the **extensive margin**. The slope of the aggregate demand curve will be affected by both sorts of decisions.



Market demand for a discrete good. The market demand curve is the sum of the demand curves of all the consumers in the market, here represented by the two consumers A and B.

We saw earlier that the adjustment on the intensive margin was in the "right" direction for normal goods: when the price went up, the quantity demanded went down. The adjustment on the extensive margin also works in the "right" direction. Thus aggregate demand curves can generally be expected to slope downward.

15.5 Elasticity

In Chapter 6 we saw how to derive a demand function from a consumer's underlying preferences. It is often of interest to have a measure of how "responsive" demand is to some change in price or income. Now the first idea that springs to mind is to use the slope of a demand function as a measure of responsiveness. After all, the definition of the slope of a demand function is the change in quantity demanded divided by the change in price:

$$\text{slope of demand function} = \frac{\Delta q}{\Delta p},$$

and that certainly looks like a measure of responsiveness.

Well, it is a measure of responsiveness—but it presents some problems. The most important one is that the slope of a demand function depends on the units in which you measure price and quantity. If you measure demand in gallons rather than in quarts, the slope becomes four times smaller. Rather than specify units all the time, it is convenient to consider a unit-free measure of responsiveness. Economists have chosen to use a measure known as **elasticity**.

The **price elasticity of demand**, ϵ, is defined to be the percent change in quantity divided by the percent change in price.[1] A 10 percent increase

[1] The Greek letter ϵ, epsilon, is pronounced "eps-i-lon."

in price is the same percentage increase whether the price is measured in American dollars or English pounds; thus measuring increases in percentage terms keeps the definition of elasticity unit-free.

In symbols the definition of elasticity is

$$\epsilon = \frac{\Delta q/q}{\Delta p/p}.$$

Rearranging this definition we have the more common expression:

$$\epsilon = \frac{p}{q}\frac{\Delta q}{\Delta p}.$$

Hence elasticity can be expressed as the ratio of price to quantity multiplied by the slope of the demand function. In the Appendix to this chapter we describe elasticity in terms of the derivative of the demand function. If you know calculus, the derivative formulation is the most convenient way to think about elasticity.

The sign of the elasticity of demand is generally negative, since demand curves invariably have a negative slope. However, it is tedious to keep referring to an elasticity of *minus* something-or-other, so it is common in verbal discussion to refer to elasticities of 2 or 3, rather than -2 or -3. We will try to keep the signs straight in the text by referring to the absolute value of elasticity, but you should be aware that verbal treatments tend to drop the minus sign.

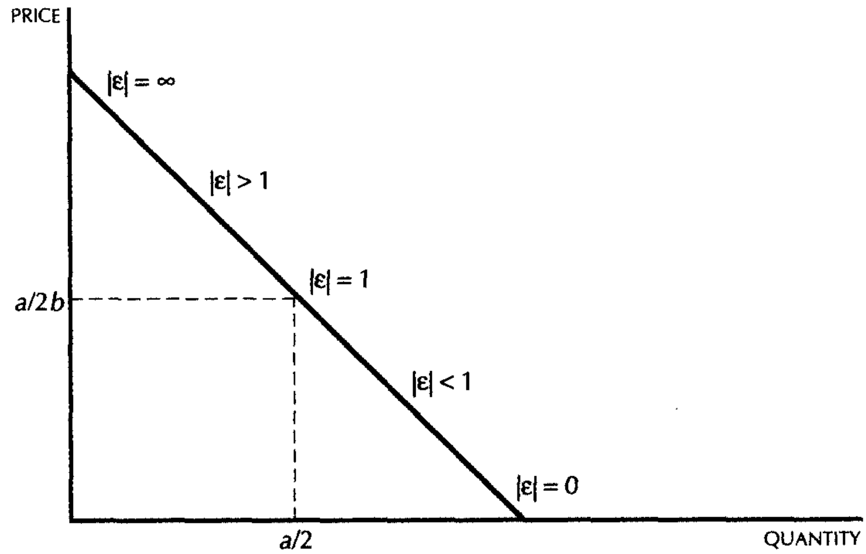
Another problem with negative numbers arises when we compare magnitudes. Is an elasticity of -3 greater or less than an elasticity of -2? From an algebraic point of view -3 is smaller than -2, but economists tend to say that the demand with the elasticity of -3 is "more elastic" than the one with -2. In this book we will make comparisons in terms of absolute value so as to avoid this kind of ambiguity.

EXAMPLE: The Elasticity of a Linear Demand Curve

Consider the linear demand curve, $q = a - bp$, depicted in Figure 15.4. The slope of this demand curve is a constant, $-b$. Plugging this into the formula for elasticity we have

$$\epsilon = \frac{-bp}{q} = \frac{-bp}{a - bp}.$$

When $p = 0$, the elasticity of demand is zero. When $q = 0$, the elasticity of demand is (negative) infinity. At what value of price is the elasticity of demand equal to -1?



The elasticity of a linear demand curve. Elasticity is infinite at the vertical intercept, one halfway down the curve, and zero at the horizontal intercept.

To find such a price, we write down the equation

$$\frac{-bp}{a - bp} = -1$$

and solve it for p. This gives

$$p = \frac{a}{2b},$$

which, as we see in Figure 15.4, is just halfway down the demand curve.

15.6 Elasticity and Demand

If a good has an elasticity of demand greater than 1 in absolute value we say that it has an **elastic demand**. If the elasticity is less than 1 in absolute value we say that it has an **inelastic demand**. And if it has an elasticity of exactly -1, we say it has **unit elastic demand**.

An elastic demand curve is one for which the quantity demanded is very responsive to price: if you increase the price by 1 percent, the quantity demanded decreases by more than 1 percent. So think of elasticity as the responsiveness of the quantity demanded to price, and it will be easy to remember what elastic and inelastic mean.

In general the elasticity of demand for a good depends to a large extent on how many close substitutes it has. Take an extreme case—our old friend, the red pencils and blue pencils example. Suppose that everyone regards these goods as perfect substitutes. Then if some of each of them are bought, they must sell for the same price. Now think what would happen to the demand for red pencils if their price rose, and the price of blue pencils stayed constant. Clearly it would drop to zero—the demand for red pencils is very elastic since it has a perfect substitute.

If a good has many close substitutes, we would expect that its demand curve would be very responsive to its price changes. On the other hand, if there are few close substitutes for a good, it can exhibit a quite inelastic demand.

15.7 Elasticity and Revenue

Revenue is just the price of a good times the quantity sold of that good. If the price of a good increases, then the quantity sold decreases, so revenue may increase or decrease. Which way it goes obviously depends on how responsive demand is to the price change. If demand drops a lot when the price increases, then revenue will fall. If demand drops only a little when the price increases, then revenue will increase. This suggests that the direction of the change in revenue has something to do with the elasticity of demand.

Indeed, there is a very useful relationship between price elasticity and revenue change. The definition of revenue is

$$R = pq.$$

If we let the price change to $p + \Delta p$ and the quantity change to $q + \Delta q$, we have a new revenue of

$$\begin{aligned} R' &= (p + \Delta p)(q + \Delta q) \\ &= pq + q\Delta p + p\Delta q + \Delta p\Delta q. \end{aligned}$$

Subtracting R from R' we have

$$\Delta R = q\Delta p + p\Delta q + \Delta p\Delta q.$$

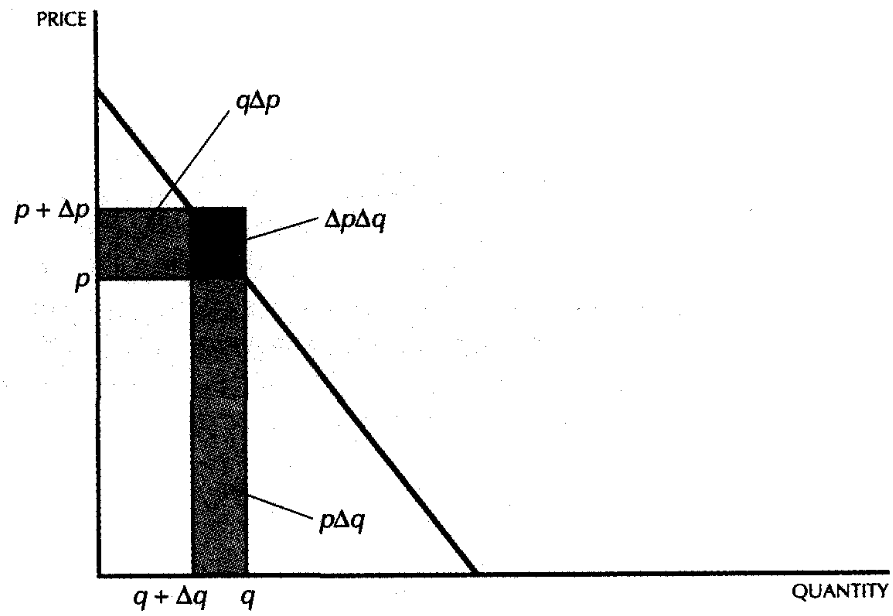
For small values of Δp and Δq, the last term can safely be neglected, leaving us with an expression for the change in revenue of the form

$$\Delta R = q\Delta p + p\Delta q.$$

That is, the change in revenue is roughly equal to the quantity times the change in price plus the original price times the change in quantity. If we want an expression for the rate of change of revenue per change in price, we just divide this expression by Δp to get

$$\frac{\Delta R}{\Delta p} = q + p\frac{\Delta q}{\Delta p}.$$

This is treated geometrically in Figure 15.5. The revenue is just the area of the box: price times quantity. When the price increases, we add a rectangular area on the top of the box, which is approximately $q\Delta p$, but we subtract an area on the side of the box, which is approximately $p\Delta q$. For small changes, this is exactly the expression given above. (The leftover part, $\Delta p \Delta q$, is the little square in the corner of the box, which will be very small relative to the other magnitudes.)



How revenue changes when price changes. The change in revenue is the sum of the box on the top minus the box on the side.

When will the net result of these two effects be positive? That is, when do we satisfy the following inequality:

$$\frac{\Delta R}{\Delta p} = p\frac{\Delta q}{\Delta p} + q(p) > 0?$$

Rearranging we have

$$\frac{p}{q}\frac{\Delta q}{\Delta p} > -1.$$

The left-hand side of this expression is $\epsilon(p)$, which is a negative number. Multiplying through by -1 reverses the direction of the inequality to give us:

$$|\epsilon(p)| < 1.$$

Thus revenue increases when price increases if the elasticity of demand is less than 1 in absolute value. Similarly, revenue decreases when price increases if the elasticity of demand is greater than 1 in absolute value.

Another way to see this is to write the revenue change as we did above:

$$\Delta R = p\Delta q + q\Delta p > 0$$

and rearrange this to get

$$-\frac{p}{q}\frac{\Delta q}{\Delta p} = |\epsilon(p)| < 1.$$

Yet a third way to see this is to take the formula for $\Delta R/\Delta p$ and rearrange it as follows:

$$\begin{aligned}\frac{\Delta R}{\Delta p} &= q + p\frac{\Delta q}{\Delta p} \\ &= q\left[1 + \frac{p}{q}\frac{\Delta q}{\Delta p}\right] \\ &= q\left[1 + \epsilon(p)\right].\end{aligned}$$

Since demand elasticity is naturally negative, we can also write this expression as

$$\frac{\Delta R}{\Delta p} = q\left[1 - |\epsilon(p)|\right].$$

In this formula it is easy to see how revenue responds to a change in price: if the absolute value of elasticity is greater than 1, then $\Delta R/\Delta p$ must be negative and vice versa.

The intuitive content of these mathematical facts is not hard to remember. If demand is very responsive to price—that is, it is very elastic—then an increase in price will reduce demand so much that revenue will fall. If demand is very unresponsive to price—it is very inelastic—then an increase in price will not change demand very much, and overall revenue will increase. The dividing line happens to be an elasticity of -1. At this point if the price increases by 1 percent, the quantity will decrease by 1 percent, so overall revenue doesn't change at all.

EXAMPLE: Strikes and Profits

In 1979 the United Farm Workers called for a strike against lettuce growers in California. The strike was highly effective: the production of lettuce was cut almost in half. But the reduction in the supply of lettuce inevitably caused an increase in the price of lettuce. In fact, during the strike the price of lettuce rose by nearly 400 percent. Since production halved and prices quadrupled, the net result of was almost a *doubling* producer profits![2]

One might well ask why the producers eventually settled the strike. The answer involves short-run and long-run supply responses. Most of the lettuce consumed in U.S. during the winter months is grown in the Imperial Valley. When the supply of this lettuce was drastically reduced in one season, there wasn't time to replace it with lettuce from elsewhere so the market price of lettuce skyrocketed. If the strike had held for several seasons, lettuce could be planted in other regions. This increase in supply from other sources would tend reduce the price of lettuce back to its normal level, thereby reducing the profits of the Imperial Valley growers.

15.8 Constant Elasticity Demands

What kind of demand curve gives us a constant elasticity of demand? In a linear demand curve the elasticity of demand goes from zero to infinity, which is not exactly what you would call constant, so that's not the answer.

We can use the revenue calculation described above to get an example. We know that if the elasticity is 1 at price p, then the revenue will not change when the price changes by a small amount. So if the revenue remains constant for all changes in price, we must have a demand curve that has an elasticity of -1 everywhere.

But this is easy. We just want price and quantity to be related by the formula

$$pq = \overline{R},$$

which means that

$$q = \frac{\overline{R}}{p}$$

is the formula for a demand function with constant elasticity of -1. The graph of the function $q = \overline{R}/p$ is given in Figure 15.6. Note that price times quantity is constant along the demand curve.

The general formula for a demand with a constant elasticity of ϵ turns out to be

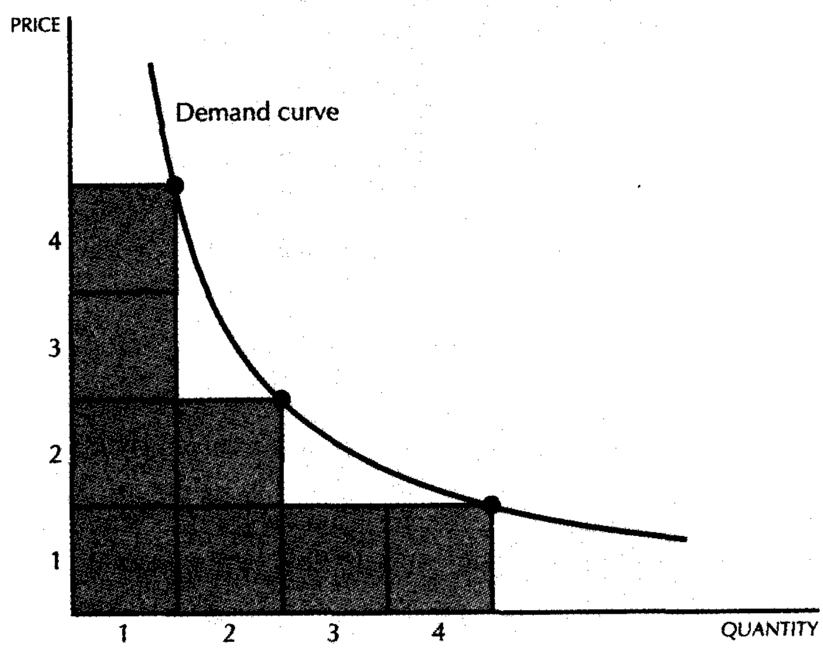
$$q = Ap^{\epsilon},$$

where A is an arbitrary positive constant and ϵ, being an elasticity, will typically be negative. This formula will be useful in some examples later on.

A convenient way to express a constant elasticity demand curve is to take logarithms and write

$$\ln q = \ln A + \epsilon \ln p.$$

[2] See Colin Carter, et. al., "Agricultural Labor Strikes and Farmers' Incomes," *Economic Inquiry*, 25, 1987,121–133.



Unit elastic demand. For this demand curve price times quantity is constant at every point. Thus the demand curve has a constant elasticity of -1.

In this expression, the logarithm of q depends in a linear way on the logarithm of p.

15.9 Elasticity and Marginal Revenue

In section 15.7 we examined how revenue changes when you change the price of a good, but it is often of interest to consider how revenue changes when you change the quantity of a good. This is especially useful when we are considering production decisions by firms.

We saw earlier that for small changes in price and quantity, the change in revenue is given by

$$\Delta R = p\Delta q + q\Delta p.$$

If we divide both sides of this expression by Δq, we get the expression for **marginal revenue**:

$$\text{MR} = \frac{\Delta R}{\Delta q} = p + q\frac{\Delta p}{\Delta q}.$$

There is a useful way to rearrange this formula. Note that we can also write this as

$$\frac{\Delta R}{\Delta q} = p\left[1 + \frac{q\Delta p}{p\Delta q}\right].$$

What is the second term inside the brackets? Nope, it's not elasticity, but you're close. It is the reciprocal of elasticity:

$$\frac{1}{\epsilon} = \frac{1}{\frac{p\Delta q}{q\Delta p}} = \frac{q\Delta p}{p\Delta q}.$$

Thus the expression for marginal revenue becomes

$$\frac{\Delta R}{\Delta q} = p(q)\left[1 + \frac{1}{\epsilon(q)}\right].$$

(Here we've written $p(q)$ and $\epsilon(q)$ to remind ourselves that both price and elasticity will typically depend on the level of output.)

When there is a danger of confusion due to the fact that elasticity is a negative number we will sometimes write this expression as

$$\frac{\Delta R}{\Delta q} = p(q)\left[1 - \frac{1}{|\epsilon(q)|}\right].$$

This means that if elasticity of demand is -1, then marginal revenue is zero—revenue doesn't change when you increase output. If demand is inelastic, then $|\epsilon|$ is less than 1, which means $1/|\epsilon|$ is greater than 1. Thus $1-1/|\epsilon|$ is negative, so that revenue will decrease when you increase output.

This is quite intuitive. If demand isn't very responsive to price, then you have to cut prices a lot to increase output: so revenue goes down. This is all completely consistent with the earlier discussion about how revenue changes as we change price, since an increase in quantity means a decrease in price and vice versa.

EXAMPLE: Setting a Price

Suppose that you were in charge of setting a price for some product that you were producing and that you had a good estimate of the demand curve for that product. Let us suppose that your goal is to set a price that maximizes profits—revenue minus costs. Then you would never want to set it where the elasticity of demand was less than 1—you would never want to set a price where demand was inelastic.

Why? Consider what would happen if you raised your price. Then your revenues would increase—since demand was inelastic—and the quantity you were selling would decrease. But if the quantity sold decreases, then your production costs must also decrease, or at least, they can't increase. So your overall profit must rise, which shows that operating at an inelastic part of the demand curve cannot yield maximal profits.

15.10 Marginal Revenue Curves

We saw in the last section that marginal revenue is given by

$$\frac{\Delta R}{\Delta q} = p(q) + \frac{\Delta p(q)}{\Delta q} q$$

or

$$\frac{\Delta R}{\Delta q} = p(q) \left[1 - \frac{1}{|\epsilon(q)|}\right].$$

We will find it useful to plot these marginal revenue curves. First, note that when quantity is zero, marginal revenue is just equal to the price. For the first unit of the good sold, the extra revenue you get is just the price. But after that, the marginal revenue will be less than the price, since $\Delta p/\Delta q$ is negative.

Think about it. If you decide to sell one more unit of output, you will have to decrease the price. But this reduction in price reduces the revenue you receive on all the units of output that you were selling already. Thus the extra revenue you receive will be less than the price that you get for selling the extra unit.

Let's consider the special case of the linear (inverse) demand curve:

$$p(q) = a - bq.$$

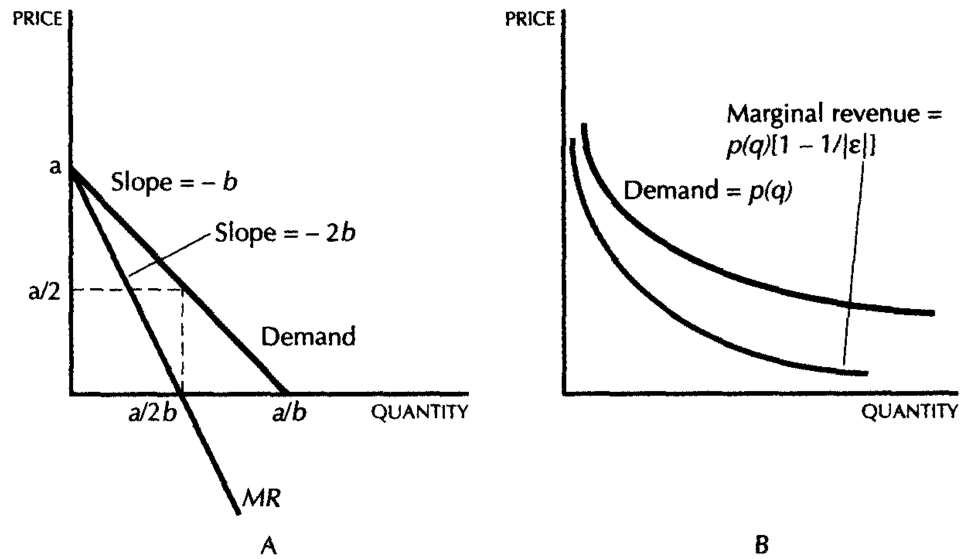
Here it is easy to see that the slope of the inverse demand curve is constant:

$$\frac{\Delta p}{\Delta q} = -b.$$

Thus the formula for marginal revenue becomes

$$\begin{aligned}\frac{\Delta R}{\Delta q} &= p(q) + \frac{\Delta p(q)}{\Delta q} q \\ &= p(q) - bq \\ &= a - bq - bq \\ &= a - 2bq.\end{aligned}$$

This marginal revenue curve is depicted in Figure 15.7A. The marginal revenue curve has the same vertical intercept as the demand curve, but has twice the slope. Marginal revenue is negative when $q > a/2b$. The quantity $a/2b$ is the quantity at which the elasticity is equal to -1. At any larger



Marginal revenue. (A) Marginal revenue for a linear demand curve. (B) Marginal revenue for a constant elasticity demand curve.

quantity demand will be inelastic, which implies that marginal revenue is negative.

The constant elasticity demand curve provides another special case of the marginal revenue curve. (See Figure 15.7B.) If the elasticity of demand is constant at $\epsilon(q) = \epsilon$, then the marginal revenue curve will have the form

$$MR = p(q)\left[1 - \frac{1}{|\epsilon|}\right].$$

Since the term in brackets is constant, the marginal revenue curve is some constant fraction of the inverse demand curve. When $|\epsilon| = 1$, the marginal revenue curve is constant at zero. When $|\epsilon| > 1$, the marginal revenue curve lies below the inverse demand curve, as depicted. When $|\epsilon| < 1$, marginal revenue is negative.

15.11 Income Elasticity

Recall that the price elasticity of demand is defined as

$$\text{price elasticity of demand} = \frac{\%\text{ change in quantity demanded}}{\%\text{ change in price}}.$$

This gives us a unit-free measure of how the amount demanded responds to a change in price.

The **income elasticity of demand** is used to describe how the quantity demanded responds to a change in income; its definition is

$$\text{income elasticity of demand} = \frac{\%\text{ change in quantity}}{\%\text{ change in income}}.$$

Recall that a **normal good** is one for which an increase in income leads to an increase in demand; so for this sort of good the income elasticity of demand is positive. An **inferior good** is one for which an increase in income leads to a decrease in demand; for this sort of good, the income elasticity of demand is negative. Economists sometimes use the term **luxury goods**. These are goods that have an income elasticity of demand that is greater than 1: a 1 percent increase in income leads to *more* than a 1 percent increase in demand for a luxury good.

As a general rule of thumb, however, income elasticities tend to cluster around 1. We can see the reason for this by examining the budget constraint. Write the budget constraints for two different levels of income:

$$p_1 x_1' + p_2 x_2' = m'$$
$$p_1 x_1^0 + p_2 x_2^0 = m^0.$$

Subtract the second equation from the first and let Δ denote differences, as usual:

$$p_1 \Delta x_1 + p_2 \Delta x_2 = \Delta m.$$

Now multiply and divide price i by x_i/x_i and divide both sides by m:

$$\frac{p_1 x_1}{m}\frac{\Delta x_1}{x_1} + \frac{p_2 x_2}{m}\frac{\Delta x_2}{x_2} = \frac{\Delta m}{m}.$$

Finally, divide both sides by $\Delta m/m$, and use $s_i = p_i x_i/m$ to denote the **expenditure share** of good i. This gives us our final equation,

$$s_1 \frac{\Delta x_1/x_1}{\Delta m/m} + s_2 \frac{\Delta x_2/x_2}{\Delta m/m} = 1.$$

This equation says that the *weighted average of the income elasticities is 1*, where the weights are the expenditure shares. Luxury goods that have an income elasticity greater than 1 must be counterbalanced by goods that have an income elasticity less than 1, so that "on average" income elasticities are about 1.

Summary

1. The market demand curve is simply the sum of the individual demand curves.

2. The reservation price measures the price at which a consumer is just indifferent between purchasing or not purchasing a good.

3. The demand function measures quantity demanded as a function of price. The inverse demand function measures price as a function of quantity. A given demand curve can be described in either way.

4. The elasticity of demand measures the responsiveness of the quantity demanded to price. It is formally defined as the percent change in quantity divided by the percent change in price.

5. If the absolute value of the elasticity of demand is less than 1 at some point, we say that demand is *inelastic* at that point. If the absolute value of elasticity is greater than 1 at some point, we say demand is *elastic* at that point. If the absolute value of the elasticity of demand at some point is exactly 1, we say that the demand has *unitary* elasticity at that point.

6. If demand is inelastic at some point, then an increase in quantity will result in a reduction in revenue. If demand is elastic, then an increase in quantity will result in an increase in revenue.

7. The marginal revenue is the extra revenue one gets from increasing the quantity sold. The formula relating marginal revenue and elasticity is $\mathrm{MR} = p[1 + 1/\epsilon] = p[1 - 1/|\epsilon|]$.

8. If the inverse demand curve is a linear function $p(q) = a - bq$, then the marginal revenue is given by $\mathrm{MR} = a - 2bq$.

9. Income elasticity measures the responsiveness of the quantity demanded to income. It is formally defined as the percent change in quantity divided by the percent change in income.

REVIEW QUESTIONS

1. If the market demand curve is $D(p) = 100 - .5p$, what is the inverse demand curve?

2. An addict's demand function for a drug may be very inelastic, but the market demand function might be quite elastic. How can this be?

3. If $D(p) = 12 - 2p$, what price will maximize revenue?

4. Suppose that the demand curve for a good is given by $D(p) = 100/p$. What price will maximize revenue?

5. True or false? In a two good model if one good is an inferior good the other good must be a luxury good.

APPENDIX

In terms of derivatives the price elasticity of demand is defined by

$$\epsilon = \frac{p}{q}\frac{dq}{dp}.$$

In the text we claimed that the formula for a constant elasticity demand curve was $q = Ap^{\epsilon}$. To verify that this is correct, we can just differentiate it with respect to price:

$$\frac{dq}{dp} = \epsilon A p^{\epsilon - 1}$$

and multiply by price over quantity:

$$\frac{p}{q}\frac{dq}{dp} = \frac{p}{Ap^{\epsilon}}\epsilon A p^{\epsilon-1} = \epsilon.$$

Everything conveniently cancels, leaving us with ϵ as required.

A linear demand curve has the formula $q(p) = a - bp$. The elasticity of demand at a point p is given by

$$\epsilon = \frac{p}{q}\frac{dq}{dp} = \frac{-bp}{a - bp}.$$

When p is zero, the elasticity is zero. When q is zero, the elasticity is infinite.

Revenue is given by $R(p) = pq(p)$. To see how revenue changes as p changes we differentiate revenue with respect to p to get

$$R'(p) = pq'(p) + q(p).$$

Suppose that revenue increases when p increases. Then we have

$$R'(p) = p\frac{dq}{dp} + q(p) > 0.$$

Rearranging, we have

$$\epsilon = \frac{p}{q}\frac{dq}{dp} > -1.$$

Recalling that dq/dp is negative and multiplying through by -1, we find

$$|\epsilon| < 1.$$

Hence if revenue increases when price increases, we must be at an inelastic part of the demand curve.

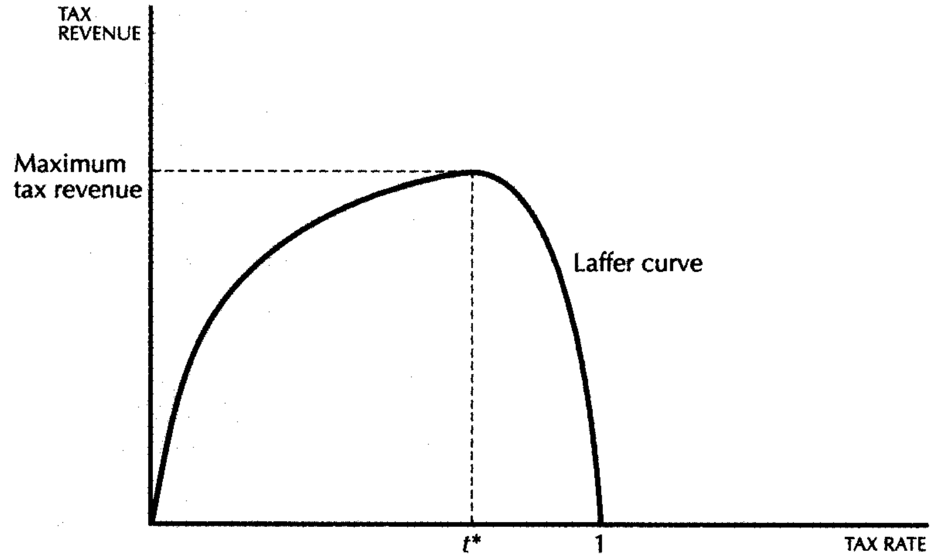


Figure 15.8 **Laffer curve.** **A possible shape for the Laffer curve, which relates tax rates and tax revenues.**

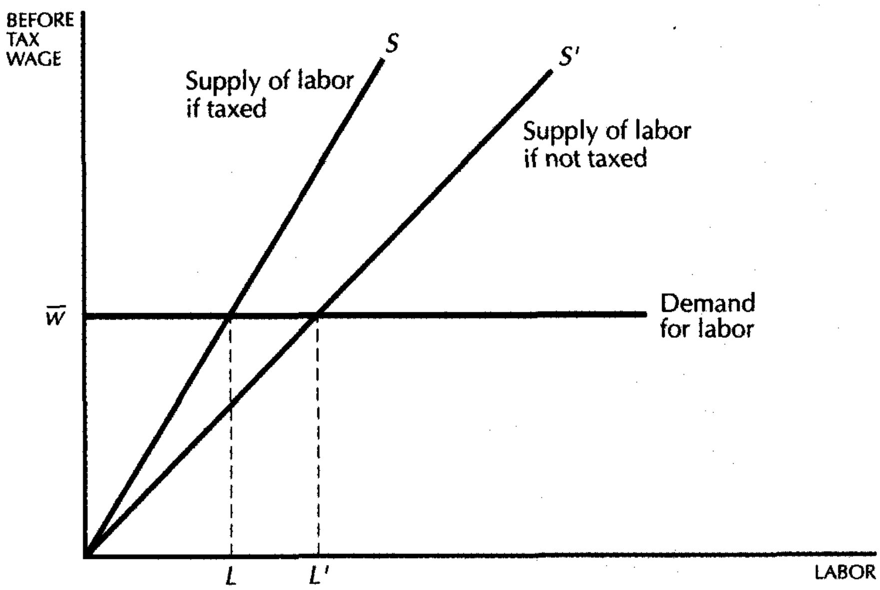
EXAMPLE: The Laffer Curve

In this section we'll consider some simple elasticity calculations that can be used to examine an issue of considerable policy interest, namely, how tax revenue changes when the tax rate changes.

Suppose that we graph tax revenue versus the tax rate. If the tax rate is zero, then tax revenues are zero; if the tax rate is 1, nobody will want to demand or supply the good in question, so the tax revenue is also zero. Thus revenue as a function of the tax rate must first increase and eventually decrease. (Of course, it can go up and down several times between zero and 1, but we'll ignore this possibility to keep things simple.) The curve that relates tax rates and tax revenues is known as the **Laffer curve**, depicted in Figure 15.8.

The interesting feature of the Laffer curve is that it suggests that when the tax rate is high enough, an increase in the tax rate will end up *reducing* the revenues collected. The reduction in the supply of the good due to the increase in the tax rate can be so large that tax revenue actually decreases. This is called the Laffer effect, after the economist who popularized this diagram in the early eighties. It has been said that the virtue of the Laffer curve is that you can explain it to a congressman in half an hour and he can talk about it for six months. Indeed, the Laffer curve figured prominently in the debate over the effect of the 1980 tax cuts. The catch in the above argument is the phrase "high enough." Just how high does the tax rate have to be for the Laffer effect to work?

To answer this question let's consider the following simple model of the labor market. Suppose that firms will demand zero labor if the wage is greater than $\overline{w}$ and an arbitrarily large amount of labor if the wage is exactly $\overline{w}$. This means that the demand curve for labor is flat at some wage $\overline{w}$. Suppose that the supply curve of labor, $S(p)$, has a conventional upward slope. The equilibrium in the labor market is depicted in Figure 15.9.



Labor market. Equilibrium in the labor market with a horizontal demand curve for labor. When labor income is taxed, less will be supplied at each wage rate.

If we put a tax on labor at the rate t, then if the firm pays $\overline{w}$, the worker only gets $w = (1 - t)\overline{w}$. Thus the supply curve of labor tilts to the left, and the amount of labor sold drops, as in Figure 15.9. The after-tax wage has gone down and this has discouraged the sale of labor. So far so good.

Tax revenue, T, is therefore given by the formula

$$T = t\overline{w}S(w),$$

where $w = (1 - t)\overline{w}$ and $S(w)$ is the supply of labor.

In order to see how tax revenue changes as we change the tax rate we differentiate this formula with respect to t to find

$$\frac{dT}{dt} = \left[-t\frac{dS(w)}{dw}\overline{w} + S(w) \right] \overline{w}. \tag{15.1}$$

(Note the use of the chain rule and the fact that $dw/dt = -\overline{w}$.)

The Laffer effect occurs when revenues decline when t increases—that is, when this expression is negative. Now this clearly means that the supply of labor is going to have to be quite elastic—it has to drop a lot when the tax increases. So let's try to see what values of elasticity will make this expression negative.

In order for equation (15.1) to be negative, we must have

$$-t\frac{dS(w)}{dw}\overline{w} + S(w) < 0.$$

Transposing yields

$$t\frac{dS(w)}{dw}\overline{w} > S(w),$$

and dividing both sides by $tS(w)$ gives

$$\frac{dS(w)}{dw}\frac{\overline{w}}{S(w)} > \frac{1}{t}.$$

Multiplying both sides by $(1-t)$ and using the fact that $w = (1-t)\overline{w}$ gives us

$$\frac{dS}{dw}\frac{w}{S} > \frac{1-t}{t}.$$

The left-hand side of this expression is the elasticity of labor supply. We have shown that the Laffer effect can only occur if the elasticity of labor supply is greater than $(1-t)/t$.

Let us take an extreme case and suppose that the tax rate on labor income is 50 percent. Then the Laffer effect can occur only when the elasticity of labor supply is greater than 1. This means that a 1 percent reduction in the wage would lead to more than a 1 percent reduction in the labor supply. This is a very large response.

Econometricians have often estimated labor-supply elasticities, and about the largest value anyone has ever found has been around 0.2. So the Laffer effect seems pretty unlikely for the kinds of tax rates that we have in the United States. However, in other countries, such as Sweden, tax rates go much higher, and there is some evidence that the Laffer phenomenon may have occurred.[3]

EXAMPLE: Another Expression for Elasticity

Here is another expression for elasticity that is sometimes useful. It turns out that elasticity can also be expressed as

$$\frac{d\ln Q}{d\ln P}.$$

The proof involves repeated application of the chain rule. We start by noting that

$$\begin{aligned}\frac{d\ln Q}{d\ln P} &= \frac{d\ln Q}{dQ}\frac{dQ}{d\ln P}\\ &= \frac{1}{Q}\frac{dQ}{d\ln P}.\end{aligned} \tag{15.2}$$

[3] See Charles E. Stuart, "Swedish Tax Rates, Labor Supply, and Tax Revenues," *Journal of Political Economy*, 89, 5 (October 1981), 1020–38.

We also note that

$$\frac{dQ}{dP} = \frac{dQ}{d\ln P}\frac{d\ln P}{dP}$$
$$= \frac{dQ}{d\ln P}\frac{1}{P},$$

which implies that

$$\frac{dQ}{d\ln P} = P\frac{dQ}{dP}.$$

Substituting this into equation (15.2), we have

$$\frac{d\ln Q}{d\ln P} = \frac{1}{Q}\frac{dQ}{dP}P = \epsilon,$$

which is what we wanted to establish.

Thus elasticity measures the slope of the demand curve plotted on log-log paper: how the log of the quantity changes as the log of the price changes.