

CHAPTER 18

TECHNOLOGY

In this chapter we begin our study of firm behavior. The first thing to do is to examine the constraints on a firm's behavior. When a firm makes choices it faces many constraints. These constraints are imposed by its customers, by its competitors, and by nature. In this chapter we're going to consider the latter source of constraints: nature. Nature imposes the constraint that there are only certain feasible ways to produce outputs from inputs: there are only certain kinds of technological choices that are possible. Here we will study how economists describe these technological constraints.

If you understand consumer theory, production theory will be very easy since the same tools are used. In fact, production theory is much simpler than consumption theory because the output of a production process is generally observable, whereas the "output" of consumption (utility) is not directly observable.

18.1 Inputs and Outputs

Inputs to production are called **factors of production**. Factors of production are often classified into broad categories such as land, labor, capital,

and raw materials. It is pretty apparent what labor, land, and raw materials mean, but capital may be a new concept. **Capital goods** are those inputs to production that are themselves produced goods. Basically capital goods are machines of one sort or another: tractors, buildings, computers, or whatever.

Sometimes capital is used to describe the money used to start up or maintain a business. We will always use the term **financial capital** for this concept and use the term capital goods, or **physical capital**, for produced factors of production.

We will usually want to think of inputs and outputs as being measured in *flow* units: a certain amount of labor per week and a certain number of machine hours per week will produce a certain amount of output a week.

We won't find it necessary to use the classifications given above very often. Most of what we want to describe about technology can be done without reference to the *kind* of inputs and outputs involved—just with the amounts of inputs and outputs.

18.2 Describing Technological Constraints

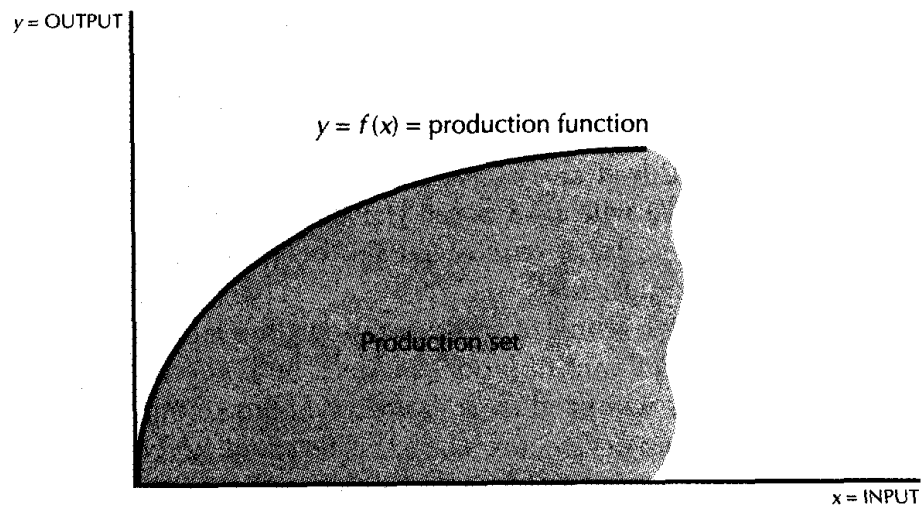
Nature imposes **technological constraints** on firms: only certain combinations of inputs are feasible ways to produce a given amount of output, and the firm must limit itself to technologically feasible production plans.

The easiest way to describe feasible production plans is to list them. That is, we can list all combinations of inputs and outputs that are technologically feasible. The set of all combinations of inputs and outputs that comprise a technologically feasible way to produce is called a **production set**.

Suppose, for example, that we have only one input, measured by x , and one output, measured by y . Then a production set might have the shape indicated in Figure 18.1. To say that some point (x, y) is in the production set is just to say that it is technologically possible to produce y amount of output if you have x amount of input. The production set shows the *possible* technological choices facing a firm.

As long as the inputs to the firm are costly it makes sense to limit ourselves to examining the *maximum possible output* for a given level of input. This is the boundary of the production set depicted in Figure 18.1. The function describing the boundary of this set is known as the **production function**. It measures the maximum possible output that you can get from a given amount of input.

Of course, the concept of a production function applies equally well if there are several inputs. If, for example, we consider the case of two inputs, the production function $f(x_1, x_2)$ would measure the maximum amount of output y that we could get if we had x_1 units of factor 1 and x_2 units of factor 2.



A production set. Here is a possible shape for a production set.

In the two-input case there is a convenient way to depict production relations known as the **isoquant**. An isoquant is the set of all possible combinations of inputs 1 and 2 that are just sufficient to produce a given amount of output.

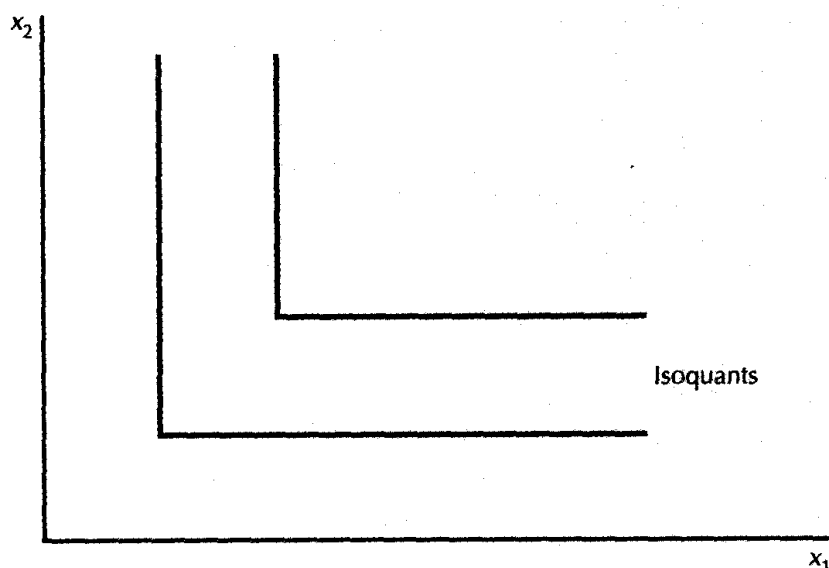
Isoquants are similar to indifference curves. As we've seen earlier, an indifference curve depicts the different consumption bundles that are just sufficient to produce a certain level of utility. But there is one important difference between indifference curves and isoquants. Isoquants are labeled with the amount of output they can produce, not with a utility level. Thus the labeling of isoquants is fixed by the technology and doesn't have the kind of arbitrary nature that the utility labeling has.

18.3 Examples of Technology

Since we already know a lot about indifference curves, it is easy to understand how isoquants work. Let's consider a few examples of technologies and their isoquants.

Fixed Proportions

Suppose that we are producing holes and that the only way to get a hole is to use one man and one shovel. Extra shovels aren't worth anything, and neither are extra men. Thus the total number of holes that you can produce will be the minimum of the number of men and the number of shovels that you have. We write the production function as $f(x_1, x_2) = \min\{x_1, x_2\}$.



Fixed proportions. Isoquants for the case of fixed proportions.

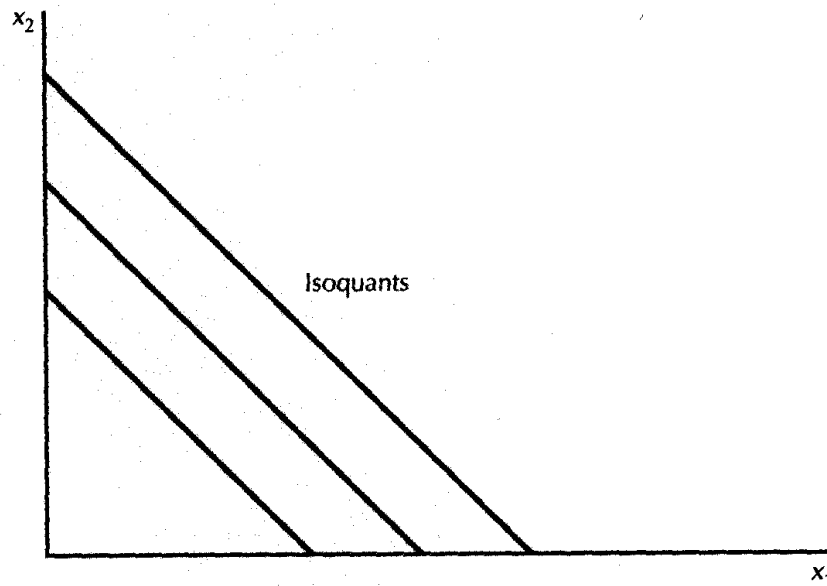
The isoquants look like those depicted in Figure 18.2. Note that these isoquants are just like the case of perfect complements in consumer theory.

Perfect Substitutes

Suppose now that we are producing homework and the inputs are red pencils and blue pencils. The amount of homework produced depends only on the total number of pencils, so we write the production function as $f(x_1, x_2) = x_1 + x_2$. The resulting isoquants are just like the case of perfect substitutes in consumer theory, as depicted in Figure 18.3.

Cobb-Douglas

If the production function has the form $f(x_1, x_2) = Ax_1^a x_2^b$, then we say that it is a **Cobb-Douglas production function**. This is just like the functional form for Cobb-Douglas preferences that we studied earlier. The numerical magnitude of the utility function was not important, so we set $A = 1$ and usually set $a + b = 1$. But the magnitude of the production function does matter so we have to allow these parameters to take arbitrary values. The parameter A measures, roughly speaking, the scale of production: how much output we would get if we used one unit of each input. The parameters a and b measure how the amount of output responds to



Perfect substitutes. Isoquants for the case of perfect substitutes.

changes in the inputs. We'll examine their impact in more detail later on. In some of the examples, we will choose to set $A = 1$ in order to simplify the calculations.

The Cobb-Douglas isoquants have the same nice, well-behaved shape that the Cobb-Douglas indifference curves have; as in the case of utility functions, the Cobb-Douglas production function is about the simplest example of well-behaved isoquants.

18.4 Properties of Technology

As in the case of consumers, it is common to assume certain properties about technology. First we will generally assume that technologies are **monotonic**: if you increase the amount of at least one of the inputs, it should be possible to produce at least as much output as you were producing originally. This is sometimes referred to as the property of **free disposal**: if the firm can costlessly dispose of any inputs, having extra inputs around can't hurt it.

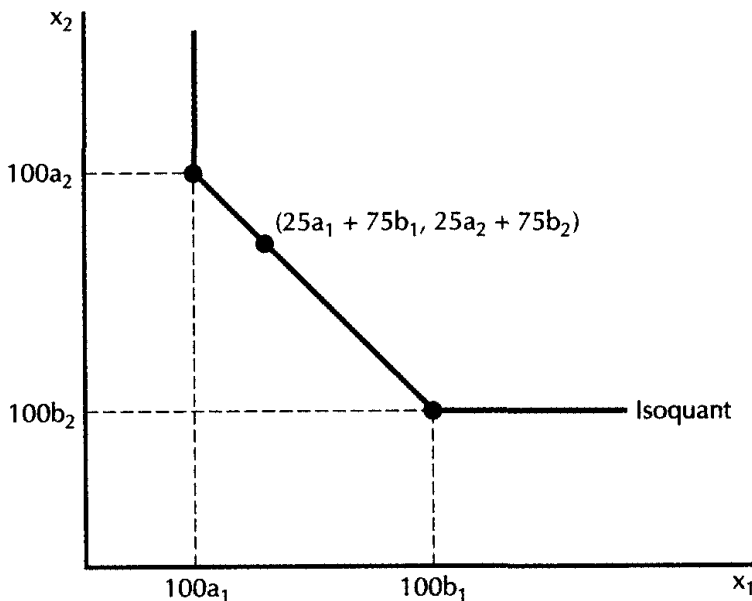
Second, we will often assume that the technology is **convex**. This means that if you have two ways to produce y units of output, (x_1, x_2) and (z_1, z_2) , then their weighted average will produce *at least* y units of output.

One argument for convex technologies goes as follows. Suppose that you have a way to produce 1 unit of output using a_1 units of factor 1 and a_2

units of factor 2 and that you have another way to produce 1 unit of output using b_1 units of factor 1 and b_2 units of factor 2. We call these two ways to produce output **production techniques**.

Furthermore, let us suppose that you are free to scale the output up by arbitrary amounts so that $(100a_1, 100a_2)$ and $(100b_1, 100b_2)$ will produce 100 units of output. But now note that if you have $25a_1 + 75b_1$ units of factor 1 and $25a_2 + 75b_2$ units of factor 2 you can still produce 100 units of output: just produce 25 units of the output using the “a” technique and 75 units of the output using the “b” technique.

This is depicted in Figure 18.4. By choosing the level at which you operate each of the two activities, you can produce a given amount of output in a variety of different ways. In particular, every input combination along the line connecting $(100a_1, 100a_2)$ and $(100b_1, 100b_2)$ will be a feasible way to produce 100 units of output.



Convexity. If you can operate production activities independently, then weighted averages of production plans will also be feasible. Thus the isoquants will have a convex shape.

In this kind of technology, where you can scale the production process up and down easily and where separate production processes don't interfere with each other, convexity is a very natural assumption.

18.5 The Marginal Product

Suppose that we are operating at some point, (x_1, x_2) , and that we consider using a little bit more of factor 1 while keeping factor 2 fixed at the level x_2 . How much more output will we get per additional unit of factor 1? We have to look at the change in output per unit change of factor 1:

$$\frac{\Delta y}{\Delta x_1} = \frac{f(x_1 + \Delta x_1, x_2) - f(x_1, x_2)}{\Delta x_1}.$$

We call this the **marginal product of factor 1**. The marginal product of factor 2 is defined in a similar way, and we denote them by $MP_1(x_1, x_2)$ and $MP_2(x_1, x_2)$, respectively.

Sometimes we will be a bit sloppy about the concept of marginal product and describe it as the extra output we get from having “one” more unit of factor 1. As long as “one” is small relative to the total amount of factor 1 that we are using, this will be satisfactory. But we should remember that a marginal product is a *rate*: the extra amount of output per unit of extra input.

The concept of marginal product is just like the concept of marginal utility that we described in our discussion of consumer theory, except for the ordinal nature of utility. Here, we are discussing physical output: the marginal product of a factor is a specific number, which can, in principle, be observed.

18.6 The Technical Rate of Substitution

Suppose that we are operating at some point (x_1, x_2) and that we consider giving up a little bit of factor 1 and using just enough more of factor 2 to produce the same amount of output y . How much extra of factor 2, Δx_2 , do we need if we are going to give up a little bit of factor 1, Δx_1 ? This is just the slope of the isoquant; we refer to it as the **technical rate of substitution (TRS)**, and denote it by $TRS(x_1, x_2)$.

The technical rate of substitution measures the tradeoff between two inputs in production. It measures the rate at which the firm will have to substitute one input for another in order to keep output constant.

To derive a formula for the TRS, we can use the same idea that we used to determine the slope of the indifference curve. Consider a change in our use of factors 1 and 2 that keeps output fixed. Then we have

$$\Delta y = MP_1(x_1, x_2)\Delta x_1 + MP_2(x_1, x_2)\Delta x_2 = 0,$$

which we can solve to get

$$TRS(x_1, x_2) = \frac{\Delta x_2}{\Delta x_1} = -\frac{MP_1(x_1, x_2)}{MP_2(x_1, x_2)}.$$

Note the similarity with the definition of the marginal rate of substitution.

18.7 Diminishing Marginal Product

Suppose that we have certain amounts of factors 1 and 2 and we consider adding more of factor 1 while holding factor 2 fixed at a given level. What might happen to the marginal product of factor 1?

As long as we have a monotonic technology, we know that the total output will go up as we increase the amount of factor 1. But it is natural to expect that it will go up at a decreasing rate. Let's consider a specific example, the case of farming.

One man on one acre of land might produce 100 bushels of corn. If we add another man and keep the same amount of land, we might get 200 bushels of corn, so in this case the marginal product of an extra worker is 100. Now keep adding workers to this acre of land. Each worker may produce more output, but eventually the extra amount of corn produced by an extra worker will be less than 100 bushels. After 4 or 5 people are added the additional output per worker will drop to 90, 80, 70 ... or even fewer bushels of corn. If we get hundreds of workers crowded together on this one acre of land, an extra worker may even cause output to go down! As in the making of broth, extra cooks *can* make things worse.

Thus we would typically expect that the marginal product of a factor will diminish as we get more and more of that factor. This is called the **law of diminishing marginal product**. It isn't really a "law"; it's just a common feature of most kinds of production processes.

It is important to emphasize that the law of diminishing marginal product applies only when all *other* inputs are being held fixed. In the farming example, we considered changing only the labor input, holding the land and raw materials fixed.

18.8 Diminishing Technical Rate of Substitution

Another closely related assumption about technology is that of **diminishing technical rate of substitution**. This says that as we increase the amount of factor 1, and adjust factor 2 so as to stay on the same isoquant, the technical rate of substitution declines. Roughly speaking, the assumption of diminishing TRS means that the slope of an isoquant must decrease in absolute value as we move along the isoquant in the direction of increasing x_1 , and it must increase as we move in the direction of increasing x_2 . This means that the isoquants will have the same sort of convex shape that well-behaved indifference curves have.

The assumptions of a diminishing technical rate of substitution and diminishing marginal product are closely related but are not exactly the same. Diminishing marginal product is an assumption about how the marginal product changes as we increase the amount of one factor, *holding the*

other factor fixed. Diminishing TRS is about how the *ratio* of the marginal products—the slope of the isoquant—changes as we increase the amount of one factor and *reduce the amount of the other factor so as to stay on the same isoquant*.

18.9 The Long Run and the Short Run

Let us return now to the original idea of a technology as being just a list of the feasible production plans. We may want to distinguish between the production plans that are *immediately* feasible and those that are *eventually* feasible.

In the **short run**, there will be some factors of production that are fixed at predetermined levels. Our farmer described above might only consider production plans that involve a fixed amount of land, if that is all he has access to. It may be true that if he had more land, he could produce more corn, but in the short run he is stuck with the amount of land that he has.

On the other hand, in the long run the farmer is free to purchase more land, or to sell some of the land he now owns. He can adjust the level of the land input so as to maximize his profits.

The economist's distinction between the long run and the short run is this: in the short run there is at least one factor of production that is fixed: a fixed amount of land, a fixed plant size, a fixed number of machines, or whatever. In the **long run**, *all* the factors of production can be varied.

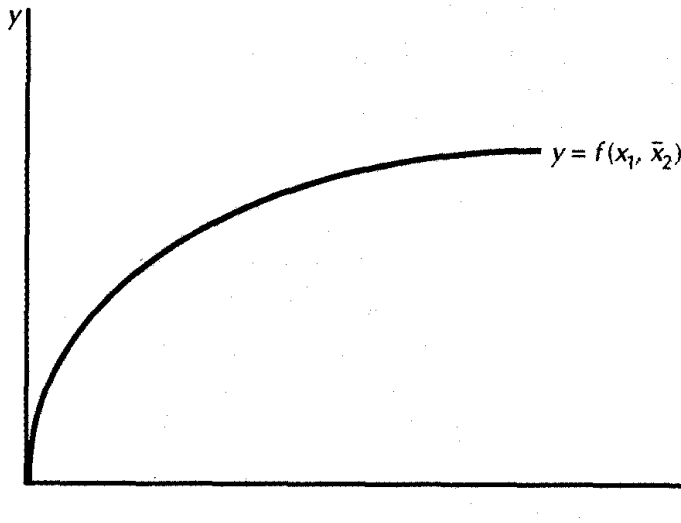
There is no specific time interval implied here. What is the long run and what is the short run depends on what kinds of choices we are examining. In the short run at least some factors are fixed at given levels, but in the long run the amount used of these factors can be changed.

Let's suppose that factor 2, say, is fixed at $\bar{x}_2$ in the short run. Then the relevant production function for the short run is $f(x_1, \bar{x}_2)$. We can plot the functional relation between output and x_1 in a diagram like Figure 18.5.

Note that we have drawn the short-run production function as getting flatter and flatter as the amount of factor 1 increases. This is just the law of diminishing marginal product in action again. Of course, it can easily happen that there is an initial region of increasing marginal returns where the marginal product of factor 1 increases as we add more of it. In the case of the farmer adding labor, it might be that the first few workers added increase output more and more because they would be able to divide up jobs efficiently, and so on. But given the fixed amount of land, eventually the marginal product of labor will decline.

18.10 Returns to Scale

Now let's consider a different kind of experiment. Instead of increasing the amount of one input while holding the other input fixed, let's increase the



Production function. This is a possible shape for a short-run production function.

amount of *all* inputs to the production function. In other words, let's scale the amount of all inputs up by some constant factor: for example, use twice as much of both factor 1 and factor 2.

If we use twice as much of each input, how much output will we get? The most likely outcome is that we will get twice as much output. This is called the case of **constant returns to scale**. In terms of the production function, this means that two times as much of each input gives two times as much output. In the case of two inputs we can express this mathematically by

$$2f(x_1, x_2) = f(2x_1, 2x_2).$$

In general, if we scale all of the inputs up by some amount t , constant returns to scale implies that we should get t times as much output:

$$tf(x_1, x_2) = f(tx_1, tx_2).$$

We say that this is the likely outcome for the following reason: it should typically be possible for the firm to *replicate* what it was doing before. If the firm has twice as much of each input, it can just set up two plants side by side and thereby get twice as much output. With three times as much of each input, it can set up three plants, and so on.

Note that it is perfectly possible for a technology to exhibit constant returns to scale and diminishing marginal product to each factor. **Returns to scale** describes what happens when you increase *all* inputs, while diminishing marginal product describes what happens when you increase *one* of the inputs and hold the others fixed.

Constant returns to scale is the most “natural” case because of the replication argument, but that isn’t to say that other things might not happen. For example, it could happen that if we scale up both inputs by some factor t , we get *more* than t times as much output. This is called the case of **increasing returns to scale**. Mathematically, increasing returns to scale means that

$$f(tx_1, tx_2) > tf(x_1, x_2).$$

for all $t > 1$.

What would be an example of a technology that had increasing returns to scale? One nice example is that of an oil pipeline. If we double the diameter of a pipe, we use twice as much materials, but the cross section of the pipe goes up by a factor of 4. Thus we will likely be able to pump more than twice as much oil through it.

(Of course, we can’t push this example too far. If we keep doubling the diameter of the pipe, it will eventually collapse of its own weight. Increasing returns to scale usually just applies over some range of output.)

The other case to consider is that of **decreasing returns to scale**, where

$$f(tx_1, tx_2) < tf(x_1, x_2)$$

for all $t > 1$.

This case is somewhat peculiar. If we get less than twice as much output from having twice as much of each input, we must be doing something wrong. After all, we could just replicate what we were doing before!

The usual way in which diminishing returns to scale arises is because we forgot to account for some input. If we have twice as much of every input but one, we won’t be able to exactly replicate what we were doing before, so there is no reason that we have to get twice as much output. Diminishing returns to scale is really a short-run phenomenon, with something being held fixed.

Of course, a technology can exhibit different kinds of returns to scale at different levels of production. It may well happen that for low levels of production, the technology exhibits increasing returns to scale—as you scale all the inputs by some small amount t , the output increases by *more* than t . Later on, for larger levels of output, increasing scale by t may just increase output by the same factor t .

Summary

1. The technological constraints of the firm are described by the production set, which depicts all the technologically feasible combinations of inputs and outputs, and by the production function, which gives the maximum amount of output associated with a given amount of the inputs.

2. Another way to describe the technological constraints facing a firm is through the use of isoquants—curves that indicate all the combinations of inputs capable of producing a given level of output.
3. We generally assume that isoquants are convex and monotonic, just like well-behaved preferences.
4. The marginal product measures the extra output per extra unit of an input, holding all other inputs fixed. We typically assume that the marginal product of an input diminishes as we use more and more of that input.
5. The technical rate of substitution (TRS) measures the slope of an isoquant. We generally assume that the TRS diminishes as we move out along an isoquant—which is another way of saying that the isoquant has a convex shape.
6. In the short run some inputs are fixed, while in the long run all inputs are variable.
7. Returns to scale refers to the way that output changes as we change the *scale* of production. If we scale all inputs up by some amount t and output goes up by the same factor, then we have constant returns to scale. If output scales up by more than t , we have increasing returns to scale; and if it scales up by less than t , we have decreasing returns to scale.

REVIEW QUESTIONS

1. Consider the production function $f(x_1, x_2) = x_1^2 x_2^2$. Does this exhibit constant, increasing, or decreasing returns to scale?
2. Consider the production function $f(x_1, x_2) = 4x_1^{\frac{1}{2}} x_2^{\frac{1}{3}}$. Does this exhibit constant, increasing, or decreasing returns to scale?
3. The Cobb-Douglas production function is given by $f(x_1, x_2) = Ax_1^a x_2^b$. It turns out that the type of returns to scale of this function will depend on the magnitude of $a + b$. Which values of $a + b$ will be associated with the different kinds of returns to scale?
4. The technical rate of substitution between factors x_2 and x_1 is -4 . If you desire to produce the same amount of output but cut your use of x_1 by 3 units, how many more units of x_2 will you need?
5. True or false? If the law of diminishing marginal product did not hold, the world's food supply could be grown in a flowerpot.
6. In a production process is it possible to have decreasing marginal product in an input and yet increasing returns to scale?