

CHAPTER 19

PROFIT MAXIMIZATION

In the last chapter we discussed ways to describe the technological choices facing the firm. In this chapter we describe a model of how the firm chooses the amount to produce and the method of production to employ. The model we will use is the model of profit maximization: the firm chooses a production plan so as to maximize its profits.

In this chapter we will assume that the firm faces fixed prices for its inputs and outputs. We said earlier that economists call a market where the individual producers take the prices as outside their control a **competitive market**. So in this chapter we want to study the profit-maximization problem of a firm that faces competitive markets for the factors of production it uses and the output goods it produces.

19.1 Profits

Profits are defined as revenues minus cost. Suppose that the firm produces n outputs $(y_1, \dots, y_n)$ and uses m inputs $(x_1, \dots, x_m)$. Let the prices of the output goods be $(p_1, \dots, p_n)$ and the prices of the inputs be $(w_1, \dots, w_m)$.

The profits the firm receives, π , can be expressed as

$$\pi = \sum_{i=1}^n p_i y_i - \sum_{i=1}^m w_i x_i.$$

The first term is revenue, and the second term is cost.

In the expression for cost we should be sure to include *all* of the factors of production used by the firm, valued at their market price. Usually this is pretty obvious, but in cases where the firm is owned and operated by the same individual, it is possible to forget about some of the factors.

For example, if an individual works in his own firm, then his labor is an input and it should be counted as part of the costs. His wage rate is simply the market price of his labor—what he *would* be getting if he sold his labor on the open market. Similarly, if a farmer owns some land and uses it in his production, that land should be valued at its market value for purposes of computing the economic costs.

We have seen that economic costs like these are often referred to as **opportunity costs**. The name comes from the idea that if you are using your labor, for example, in one application, you forgo the opportunity of employing it elsewhere. Therefore those lost wages are part of the cost of production. Similarly with the land example: the farmer has the opportunity of renting his land to someone else, but he chooses to forgo that rental income in favor of renting it to himself. The lost rents are part of the opportunity cost of his production.

The economic definition of profit requires that we value all inputs and outputs at their opportunity cost. Profits as determined by accountants do not necessarily accurately measure economic profits, as they typically use historical costs—what a factor was purchased for originally—rather than economic costs—what a factor would cost if purchased now. There are many variations on the use of the term “profit,” but we will always stick to the economic definition.

Another confusion that sometimes arises is due to getting time scales mixed up. We usually think of the factor inputs as being measured in terms of *flows*. So many labor hours per week and so many machine hours per week will produce so much output per week. Then the factor prices will be measured in units appropriate for the purchase of such flows. Wages are naturally expressed in terms of dollars per hour. The analog for machines would be the **rental rate**—the rate at which you can rent a machine for the given time period.

In many cases there isn't a very well-developed market for the rental of machines, since firms will typically buy their capital equipment. In this case, we have to compute the implicit rental rate by seeing how much it would cost to buy a machine at the beginning of the period and sell it at the end of the period.

19.2 The Organization of Firms

In a capitalist economy, firms are owned by individuals. Firms are only legal entities; ultimately it is the owners of firms who are responsible for the behavior of the firm, and it is the owners who reap the rewards or pay the costs of that behavior.

Generally speaking, firms can be organized as proprietorships, partnerships, or corporations. A **proprietorship** is a firm that is owned by a single individual. A **partnership** is owned by two or more individuals. A **corporation** is usually owned by several individuals as well, but under the law has an existence separate from that of its owners. Thus a partnership will last only as long as both partners are alive and agree to maintain its existence. A corporation can last longer than the lifetimes of any of its owners. For this reason, most large firms are organized as corporations.

The owners of each of these different types of firms may have different goals with respect to managing the operation of the firm. In a proprietorship or a partnership the owners of the firm usually take a direct role in actually managing the day-to-day operations of the firm, so they are in a position to carry out whatever objectives they have in operating the firm. Typically, the owners would be interested in maximizing the profits of their firm, but, if they have nonprofit goals, they can certainly indulge in these goals instead.

In a corporation, the owners of the corporation are often distinct from the managers of the corporation. Thus there is a separation of ownership and control. The owners of the corporation must define an objective for the managers to follow in their running of the firm, and then do their best to see that they actually pursue the goals the owners have in mind. Again, profit maximization is a common goal. As we'll see below, this goal, properly interpreted, is likely to lead the managers of the firm to choose actions that are in the interests of the owners of the firm.

19.3 Profits and Stock Market Value

Often the production process that a firm uses goes on for many periods. Inputs put in place at time t pay off with a whole flow of services at later times. For example, a factory building erected by a firm could last for 50 or 100 years. In this case an input at one point in time helps to produce output at other times in the future.

In this case we have to value a flow of costs and a flow of revenues over time. As we've seen in Chapter 10, the appropriate way to do this is to use the concept of present value. When people can borrow and lend in financial markets, the interest rate can be used to define a natural price of consumption at different times. Firms have access to the same sorts of

financial markets, and the interest rate can be used to value investment decisions in exactly the same way.

Consider a world of perfect certainty where a firm's flow of future profits is publicly known. Then the present value of those profits would be the **present value of the firm**. It would be how much someone would be willing to pay to purchase the firm.

As we indicated above, most large firms are organized as corporations, which means that they are jointly owned by a number of individuals. The corporation issues stock certificates to represent ownership of shares in the corporation. At certain times the corporation issues dividends on these shares, which represent a share of the profits of the firm. The shares of ownership in the corporation are bought and sold in the **stock market**. The price of a share represents the present value of the stream of dividends that people expect to receive from the corporation. The total stock market value of a firm represents the present value of the stream of profits that the firm is expected to generate. Thus the objective of the firm—maximizing the present value of the stream of profits the firm generates—could also be described as the goal of maximizing stock market value. In a world of certainty, these two goals are the same thing.

The owners of the firm will generally want the firm to choose production plans that maximize the stock market value of the firm, since that will make the value of the shares they hold as large as possible. We saw in Chapter 10 that whatever an individual's tastes for consumption at different times, he or she will always prefer an endowment with a higher present value to one with a lower present value. By maximizing stock market value, a firm makes its shareholders' budget sets as large as possible, and thereby acts in the best interests of all of its shareholders.

If there is uncertainty about a firm's stream of profits, then instructing managers to maximize profits has no meaning. Should they maximize expected profits? Should they maximize the expected utility of profits? What attitude toward risky investments should the managers have? It is difficult to assign a meaning to profit maximization when there is uncertainty present. However, in a world of uncertainty, maximizing *stock market value* still has meaning. If the managers of a firm attempt to make the value of the firm's shares as large as possible then they make the firm's owners—the shareholders—as well-off as possible. Thus maximizing stock market value gives a well-defined objective function to the firm in nearly all economic environments.

Despite these remarks about time and uncertainty, we will generally limit ourselves to the examination of much simpler profit-maximization problems, namely, those in which there is a single, certain output and a single period of time. This simple story still generates significant insights and builds the proper intuition to study more general models of firm behavior. Most of the ideas that we will examine carry over in a natural way to these more general models.

19.4 The Boundaries of the Firm

One question that constantly confronts managers of firms is whether to “make or buy.” That is, should a firm make something internally or buy it from an external supplier? The question is broader than it sounds, as it can refer not only to physical goods, but also services of one sort or another. Indeed, in the broadest interpretation, “make or buy” applies to almost every decision a firm makes.

Should a company provide its own cafeteria? Janitorial services? Photocopying services? Travel assistance? Obviously, many factors enter into such decisions. One important consideration is size. A small mom-and-pop video store with 12 employees is probably not going to provide a cafeteria. But it might outsource janitorial services, depending on cost, capabilities, and staffing.

Even a large organization, which could easily afford to operate food services, may or may not choose to do so, depending on availability of alternatives. Employees of an organization located in a big city have access to many places to eat; if the organization is located in a remote area, choices may be fewer.

One critical issue is whether the goods or services in question are externally provided by a monopoly or by a competitive market. By and large, managers prefer to buy goods and services on a competitive market, if they are available. The second-best choice is dealing with an internal monopolist. The worse choice of all, in terms of price and quality of service, is dealing with an external monopolist.

Think about photocopying services. The ideal situation is to have dozens of competitive providers vying for your business; that way you will get cheap prices and high-quality service. If your school is large, or in an urban area, there may be many photocopying services vying for your business. On the other hand, small rural schools may have less choice and often higher prices.

The same is true of businesses. A highly competitive environment gives lots of choices to users. By comparison, an internal photocopying division may be less attractive. Even if prices are low, the service could be sluggish. But the least attractive option is surely to have to submit to a single external provider. An internal monopoly provider may have bad service, but at least the money stays inside the firm.

As technology changes, what is typically inside the firm changes. Forty years ago, firms managed many services themselves. Now they tend to outsource as much as possible. Food service, photocopying service, and janitorial services are often provided by external organizations that specialize in such activities. Such specialization often allows these companies to provide higher quality and less expensive services to the organizations that use their services.

19.5 Fixed and Variable Factors

In a given time period, it may be very difficult to adjust some of the inputs. Typically a firm may have contractual obligations to employ certain inputs at certain levels. An example of this would be a lease on a building, where the firm is legally obligated to purchase a certain amount of space over the period under examination. We refer to a factor of production that is in a fixed amount for the firm as a **fixed factor**. If a factor can be used in different amounts, we refer to it as a **variable factor**.

As we saw in Chapter 18, the short run is defined as that period of time in which there are some fixed factors—factors that can only be used in fixed amounts. In the long run, on the other hand, the firm is free to vary all of the factors of production: all factors are variable factors.

There is no rigid boundary between the short run and the long run. The exact time period involved depends on the problem under examination. The important thing is that some of the factors of production are fixed in the short run and variable in the long run. Since all factors are variable in the long run, a firm is always free to decide to use zero inputs and produce zero output—that is, to go out of business. Thus the least profits a firm can make in the long run are zero profits.

In the short run, the firm is obligated to employ some factors, even if it decides to produce zero output. Therefore it is perfectly possible that the firm could make *negative* profits in the short run.

By definition, fixed factors are factors of production that must be paid for even if the firm decides to produce zero output: if a firm has a long-term lease on a building, it must make its lease payments each period whether or not it decides to produce anything that period. But there is another category of factors that only need to be paid for if the firm decides to produce a positive amount of output. One example is electricity used for lighting. If the firm produces zero output, it doesn't have to provide any lighting; but if it produces any positive amount of output, it has to purchase a fixed amount of electricity to use for lighting.

Factors such as these are called **quasi-fixed factors**. They are factors of production that must be used in a fixed amount, independent of the output of the firm, as long as the output is positive. The distinction between fixed factors and quasi-fixed factors is sometimes useful in analyzing the economic behavior of the firm.

19.6 Short-Run Profit Maximization

Let's consider the short-run profit-maximization problem when input 2 is fixed at some level $\bar{x}_2$. Let $f(x_1, x_2)$ be the production function for the firm, let p be the price of output, and let w_1 and w_2 be the prices of the

two inputs. Then the profit-maximization problem facing the firm can be written as

$$\max_{x_1} pf(x_1, \bar{x}_2) - w_1x_1 - w_2\bar{x}_2.$$

The condition for the optimal choice of factor 1 is not difficult to determine.

If x_1^* is the profit-maximizing choice of factor 1, then the output price times the marginal product of factor 1 should equal the price of factor 1. In symbols,

$$pMP_1(x_1^*, \bar{x}_2) = w_1.$$

In other words, the *value of the marginal product of a factor should equal its price*.

In order to understand this rule, think about the decision to employ a little more of factor 1. As you add a little more of it, Δx_1 , you produce $\Delta y = MP_1 \Delta x_1$ more output that is worth $pMP_1 \Delta x_1$. But this marginal output costs $w_1 \Delta x_1$ to produce. If the value of marginal product exceeds its cost, then profits can be increased by *increasing* input 1. If the value of marginal product is less than its cost, then profits can be increased by *decreasing* the level of input 1.

If the profits of the firm are as large as possible, then profits should not increase when we increase or decrease input 1. This means that at a profit-maximizing choice of inputs and outputs, the value of the marginal product, $pMP_1(x_1^*, \bar{x}_2)$, should equal the factor price, w_1 .

We can derive the same condition graphically. Consider Figure 19.1. The curved line represents the production function holding factor 2 fixed at $\bar{x}_2$. Using y to denote the output of the firm, profits are given by

$$\pi = py - w_1x_1 - w_2\bar{x}_2.$$

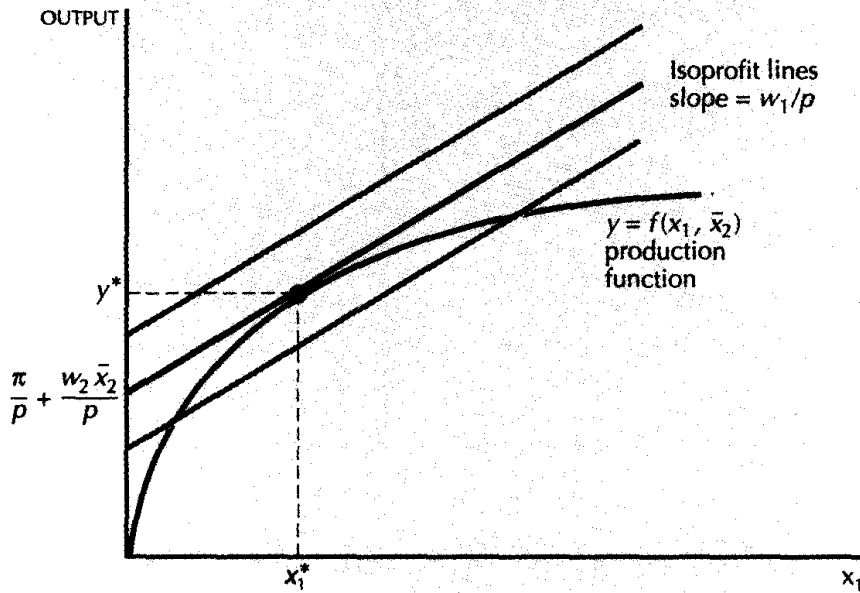
This expression can be solved for y to express output as a function of x_1 :

$$y = \frac{\pi}{p} + \frac{w_2}{p}\bar{x}_2 + \frac{w_1}{p}x_1. \quad (19.1)$$

This equation describes **isoprofit lines**. These are just all combinations of the input goods and the output good that give a constant level of profit, π . As π varies we get a family of parallel straight lines each with a slope of w_1/p and each having a vertical intercept of $\pi/p + w_2\bar{x}_2/p$, which measures the profits plus the fixed costs of the firm.

The fixed costs are fixed, so the only thing that really varies as we move from one isoprofit line to another is the level of profits. Thus higher levels of profit will be associated with isoprofit lines with higher vertical intercepts.

The profit-maximization problem is then to find the point on the production function that has the highest associated isoprofit line. Such a point is illustrated in Figure 19.1. As usual it is characterized by a tangency condition: the slope of the production function should equal the slope of



Profit maximization. The firm chooses the input and output combination that lies on the highest isoprofit line. In this case the profit-maximizing point is (x_1^*, y^*) .

the isoprofit line. Since the slope of the production function is the marginal product, and the slope of the isoprofit line is w_1/p , this condition can also be written as

$$MP_1 = \frac{w_1}{p},$$

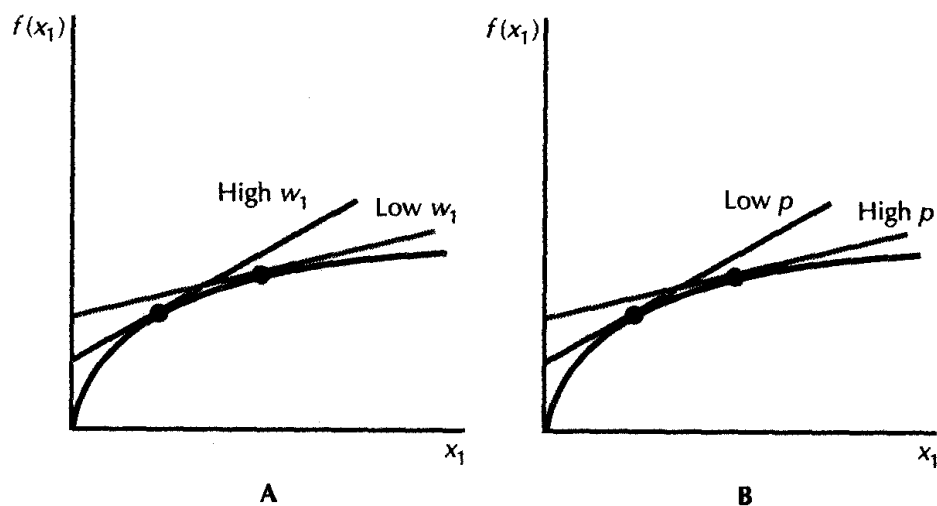
which is equivalent to the condition we derived above.

19.7 Comparative Statics

We can use the geometry depicted in Figure 19.1 to analyze how a firm's choice of inputs and outputs varies as the prices of inputs and outputs vary. This gives us one way to analyze the **comparative statics** of firm behavior.

For example: how does the optimal choice of factor 1 vary as we vary its factor price w_1 ? Referring to equation (19.1), which defines the isoprofit line, we see that increasing w_1 will make the isoprofit line steeper, as shown in Figure 19.2A. When the isoprofit line is steeper, the tangency must occur further to the left. Thus the optimal level of factor 1 must decrease. This simply means that as the price of factor 1 increases, the demand for factor 1 must decrease: factor demand curves must slope downward.

Similarly, if the output price decreases the isoprofit line must become steeper, as shown in Figure 19.2B. By the same argument as given in the



Comparative statics. Panel A shows that increasing w_1 will reduce the demand for factor 1. Panel B shows that increasing the price of output will increase the demand for factor 1 and therefore increase the supply of output.

last paragraph the profit-maximizing choice of factor 1 will decrease. If the amount of factor 1 decreases and the level of factor 2 is fixed in the short run by assumption, then the supply of output must decrease. This gives us another comparative statics result: a reduction in the output price must decrease the supply of output. In other words, the supply function must slope upwards.

Finally, we can ask what will happen if the price of factor 2 changes? Because this is a short-run analysis, changing the price of factor 2 will not change the firm's choice of factor 2—in the short run, the level of factor 2 is fixed at $\bar{x}_2$. Changing the price of factor 2 has no effect on the *slope* of the isoprofit line. Thus the optimal choice of factor 1 will not change, nor will the supply of output. All that changes are the profits that the firm makes.

19.8 Profit Maximization in the Long Run

In the long run the firm is free to choose the level of all inputs. Thus the long-run profit-maximization problem can be posed as

$$\max_{x_1, x_2} pf(x_1, x_2) - w_1x_1 - w_2x_2.$$

This is basically the same as the short-run problem described above, but now both factors are free to vary.

The condition describing the optimal choices is essentially the same as before, but now we have to apply it to *each* factor. Before we saw that the value of the marginal product of factor 1 must be equal to its price, whatever the level of factor 2. The same sort of condition must now hold for *each* factor choice:

$$pMP_1(x_1^*, x_2^*) = w_1$$

$$pMP_2(x_1^*, x_2^*) = w_2.$$

If the firm has made the optimal choices of factors 1 and 2, the value of the marginal product of each factor should equal its price. At the optimal choice, the firm's profits cannot increase by changing the level of either input.

The argument is the same as used for the short-run profit-maximizing decisions. If the value of the marginal product of factor 1, for example, exceeded the price of factor 1, then using a little more of factor 1 would produce MP_1 more output, which would sell for pMP_1 dollars. If the value of this output exceeds the cost of the factor used to produce it, it clearly pays to expand the use of this factor.

These two conditions give us two equations in two unknowns, x_1^* and x_2^* . If we know how the marginal products behave as a function of x_1 and x_2 , we will be able to solve for the optimal choice of each factor as a function of the prices. The resulting equations are known as the **factor demand curves**.

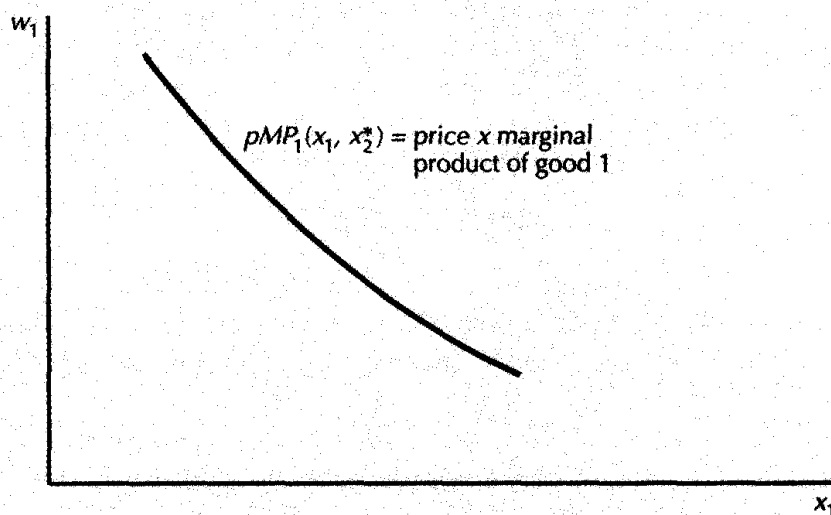
19.9 Inverse Factor Demand Curves

The **factor demand curves** of a firm measure the relationship between the price of a factor and the profit-maximizing choice of that factor. We saw above how to find the profit-maximizing choices: for any prices, (p, w_1, w_2) , we just find those factor demands, (x_1^*, x_2^*) , such that the value of the marginal product of each factor equals its price.

The **inverse factor demand curve** measures the same relationship, but from a different point of view. It measures what the factor prices must be for some given quantity of inputs to be demanded. Given the optimal choice of factor 2, we can draw the relationship between the optimal choice of factor 1 and its price in a diagram like that depicted in Figure 19.3. This is simply a graph of the equation

$$pMP_1(x_1, x_2^*) = w_1.$$

This curve will be downward sloping by the assumption of diminishing marginal product. For any level of x_1 , this curve depicts what the factor price must be in order to induce the firm to demand that level of x_1 , holding factor 2 fixed at x_2^* .



The inverse factor demand curve. This measures what the price of factor 1 must be to get x_1 units demanded if the level of the other factor is held fixed at x_2^* .

19.10 Profit Maximization and Returns to Scale

There is an important relationship between competitive profit maximization and returns to scale. Suppose that a firm has chosen a long-run profit-maximizing output $y^* = f(x_1^*, x_2^*)$, which it is producing using input levels (x_1^*, x_2^*) .

Then its profits are given by

$$\pi^* = py^* - w_1x_1^* - w_2x_2^*.$$

Suppose that this firm's production function exhibits constant returns to scale and that it is making positive profits in equilibrium. Then consider what would happen if it doubled the level of its input usage. According to the constant returns to scale hypothesis, it would double its output level. What would happen to profits?

It is not hard to see that its profits would also double. But this contradicts the assumption that its original choice was profit maximizing! We derived this contradiction by assuming that the original profit level was positive; if the original level of profits were zero there would be no problem: two times zero is still zero.

This argument shows that the only reasonable long-run level of profits for a competitive firm that has constant returns to scale at all levels of output is a zero level of profits. (Of course if a firm has negative profits in the long run, it should go out of business.)

Most people find this to be a surprising statement. Firms are out to maximize profits aren't they? How can it be that they can only get zero profits in the long run?

Think about what would happen to a firm that did try to expand indefinitely. Three things might occur. First, the firm could get so large that it could not really operate effectively. This is just saying that the firm *really* doesn't have constant returns to scale at all levels of output. Eventually, due to coordination problems, it might enter a region of decreasing returns to scale.

Second, the firm might get so large that it would totally dominate the market for its product. In this case there is no reason for it to behave competitively—to take the price of output as given. Instead, it would make sense for such a firm to try to use its size to influence the market price. The model of competitive profit maximization would no longer be a sensible way for the firm to behave, since it would effectively have no competitors. We'll investigate more appropriate models of firm behavior in this situation when we discuss monopoly.

Third, if one firm can make positive profits with a constant returns to scale technology, so can any other firm with access to the same technology. If one firm wants to expand its output, so would other firms. But if all firms expand their outputs, this will certainly push down the price of output and lower the profits of all the firms in the industry.

19.11 Revealed Profitability

When a profit-maximizing firm makes its choice of inputs and outputs it reveals two things: first, that the inputs and outputs used represent a *feasible* production plan, and second, that these choices are more profitable than other feasible choices that the firm could have made. Let us examine these points in more detail.

Suppose that we observe two choices that the firm makes at two different sets of prices. At time t , it faces prices (p^t, w_1^t, w_2^t) and makes choices (y^t, x_1^t, x_2^t) . At time s , it faces prices (p^s, w_1^s, w_2^s) and makes choices (y^s, x_1^s, x_2^s) . If the production function of the firm hasn't changed between times s and t and if the firm is a profit maximizer, then we must have

$$p^t y^t - w_1^t x_1^t - w_2^t x_2^t \geq p^t y^s - w_1^t x_1^s - w_2^t x_2^s \quad (19.2)$$

and

$$p^s y^s - w_1^s x_1^s - w_2^s x_2^s \geq p^s y^t - w_1^s x_1^t - w_2^s x_2^t. \quad (19.3)$$

That is, the profits that the firm achieved facing the t period prices must be larger than if they used the s period plan and vice versa. If either of these inequalities were violated, the firm could not have been a profit-maximizing firm (with an unchanging technology).

Thus if we ever observe two time periods where these inequalities are violated we would know that the firm was not maximizing profits in at least one of the two periods. The satisfaction of these inequalities is virtually an axiom of profit-maximizing behavior, so it might be referred to as the **Weak Axiom of Profit Maximization (WAPM)**.

If the firm's choices satisfy WAPM, we can derive a useful comparative statics statement about the behavior of factor demands and output supplies when prices change. Transpose the two sides of equation (19.3) to get

$$-p^s y^t + w_1^s x_1^t + w_2^s x_1^t \geq -p^s y^s + w_1^s x_1^s + w_2^s x_2^s \quad (19.4)$$

and add equation (19.4) to equation (19.2) to get

$$\begin{aligned} & (p^t - p^s)y^t - (w_1^t - w_1^s)x_1^t - (w_2^t - w_2^s)x_2^t \\ & \geq (p^t - p^s)y^s - (w_1^t - w_1^s)x_1^s - (w_2^t - w_2^s)x_2^s. \end{aligned} \quad (19.5)$$

Now rearrange this equation to yield

$$(p^t - p^s)(y^t - y^s) - (w_1^t - w_1^s)(x_1^t - x_1^s) - (w_2^t - w_2^s)(x_2^t - x_2^s) \geq 0. \quad (19.6)$$

Finally define the change in prices, $\Delta p = (p^t - p^s)$, the change in output, $\Delta y = (y^t - y^s)$, and so on to find

$$\Delta p \Delta y - \Delta w_1 \Delta x_1 - \Delta w_2 \Delta x_2 \geq 0. \quad (19.7)$$

This equation is our final result. It says that the change in the price of output times the change in output minus the change in each factor price times the change in that factor must be nonnegative. This equation comes solely from the definition of profit maximization. Yet it contains all of the comparative statics results about profit-maximizing choices!

For example, suppose that we consider a situation where the price of output changes, but the price of each factor stays constant. If $\Delta w_1 = \Delta w_2 = 0$, then equation (19.7) reduces to

$$\Delta p \Delta y \geq 0.$$

Thus if the price of output goes up, so that $\Delta p > 0$, then the change in output must be nonnegative as well, $\Delta y \geq 0$. This says that the profit-maximizing supply curve of a competitive firm must have a positive (or at least a zero) slope.

Similarly, if the price of output and of factor 2 remain constant, equation (19.7) becomes

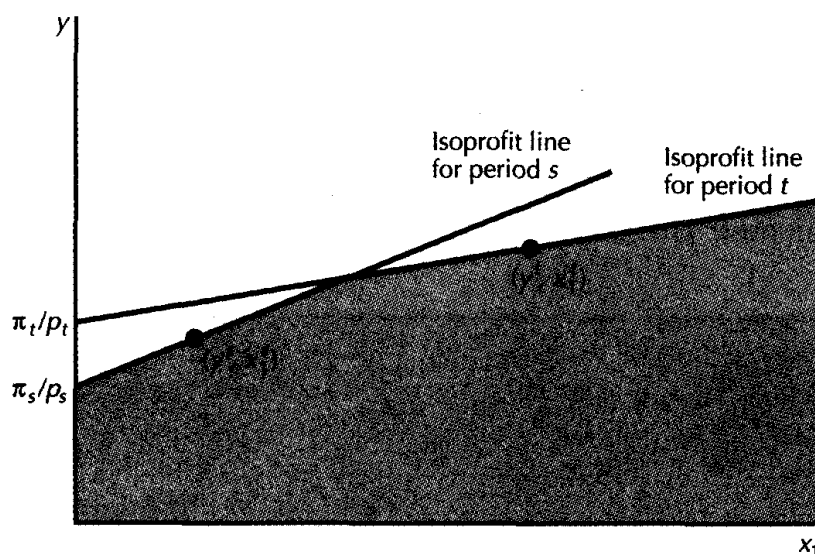
$$-\Delta w_1 \Delta x_1 \geq 0,$$

which is to say

$$\Delta w_1 \Delta x_1 \leq 0.$$

Thus if the price of factor 1 goes up, so that $\Delta w_1 > 0$, then equation (19.7) implies that the demand for factor 1 will go down (or at worst stay the same), so that $\Delta x_1 \leq 0$. This means that the factor demand curve must be a decreasing function of the factor price: factor demand curves have a negative slope.

The simple inequality in WAPM, and its implication in equation (19.7), give us strong observable restrictions about how a firm will behave. It is natural to ask whether these are all of the restrictions that the model of profit maximization imposes on firm behavior. Said another way, if we observe a firm's choices, and these choices satisfy WAPM, can we construct an estimate of the technology for which the observed choices are profit-maximizing choices? It turns out that the answer is yes. Figure 19.4 shows how to construct such a technology.



Construction of a possible technology. If the observed choices are maximal profit choices at each set of prices, then we can estimate the shape of the technology that generated those choices by using the isoprofit lines.

In order to illustrate the argument graphically, we suppose that there is one input and one output. Suppose that we are given an observed choice in period t and in period s , which we indicate by (p^t, w_1^t, y^t, x_1^t) and (p^s, w_1^s, y^s, x_1^s) . In each period we can calculate the profits π_s and π_t and plot all the combinations of y and x_1 that yield these profits.

That is, we plot the two isoprofit lines

$$\pi_t = p^t y - w_1^t x_1$$

and

$$\pi_s = p^s y - w_1^s x_1.$$

The points above the isoprofit line for period t have higher profits than π_t at period t prices, and the points above the isoprofit line for period s have higher profits than π_s at period s prices. WAPM requires that the choice in period t must lie below the period s isoprofit line and that the choice in period s must lie below the period t isoprofit line.

If this condition is satisfied, it is not hard to generate a technology for which (y^t, x_1^t) and (y^s, x_1^s) are profit-maximizing choices. Just take the shaded area beneath the two lines. These are all of the choices that yield lower profits than the observed choices at both sets of prices.

The proof that this technology will generate the observed choices as profit-maximizing choices is clear geometrically. At the prices (p^t, w_1^t) , the choice (y^t, x_1^t) is on the highest isoprofit line possible, and the same goes for the period s choice.

Thus, when the observed choices satisfy WAPM, we can “reconstruct” an estimate of a technology that might have generated the observations. In this sense, any observed choices consistent with WAPM could be profit-maximizing choices. As we observe more choices that the firm makes, we get a tighter estimate of the production function, as illustrated in Figure 19.5.

This estimate of the production function can be used to forecast firm behavior in other environments or for other uses in economic analysis.

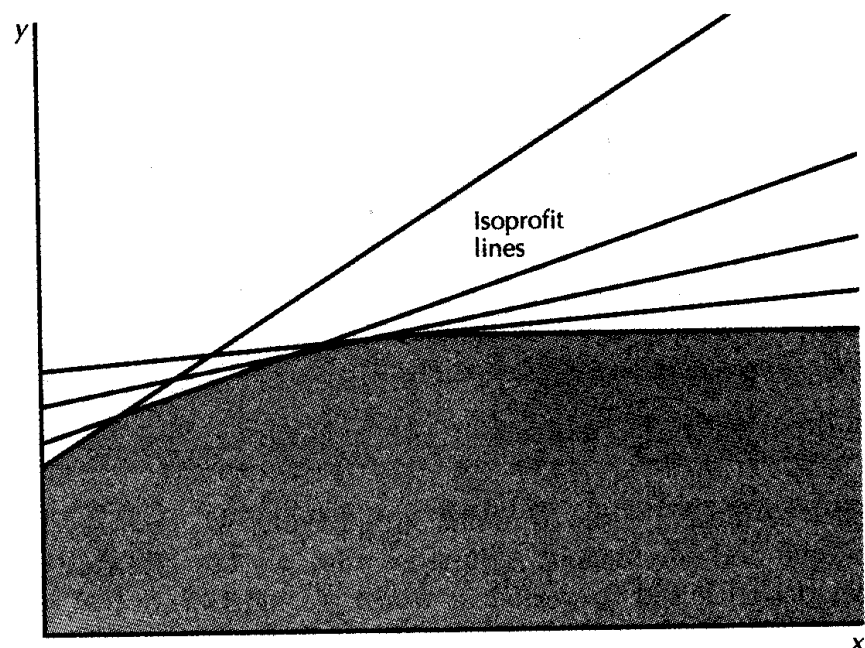


Figure
19.5

Estimating the technology. As we observe more choices we get a tighter estimate of the production function.

EXAMPLE: How Do Farmers React to Price Supports?

The U.S. government currently spends between \$40 and \$60 billion a year in aid to farmers. A large fraction of this amount is used to subsidize the production of various products including milk, wheat, corn, soybeans, and cotton. Occasionally, attempts are made to reduce or eliminate these subsidies. The effect of elimination of these subsidies would be to reduce the price of the product received by the farmers.

Farmers sometimes argue that eliminating the subsidies to milk, for example, would not reduce the total supply of milk, since dairy farmers would choose to *increase* their herds and their supply of milk so as to keep their standard of living constant.

If farmers are behaving so as to maximize profits, this is impossible. As we've seen above, the logic of profit maximization *requires* that a decrease in the price of an output leads to a reduction in its supply: if Δp is negative, then Δy must be negative as well.

It is certainly possible that small family farms have goals other than simple maximization of profits, but larger "agribusiness" farms are more likely to be profit maximizers. Thus the perverse response to the elimination of subsidies alluded to above could only occur on a limited scale, if at all.

19.12 Cost Minimization

If a firm is maximizing profits and if it chooses to supply some output y , then it must be minimizing the cost of producing y . If this were not so, then there would be some cheaper way of producing y units of output, which would mean that the firm was not maximizing profits in the first place.

This simple observation turns out to be quite useful in examining firm behavior. It turns out to be convenient to break the profit-maximization problem into two stages: first we figure out how to minimize the costs of producing any desired level of output y , then we figure out which level of output is indeed a profit-maximizing level of output. We begin this task in the next chapter.

Summary

1. Profits are the difference between revenues and costs. In this definition it is important that all costs be measured using the appropriate market prices.
2. Fixed factors are factors whose amount is independent of the level of output; variable factors are factors whose amount used changes as the level of output changes.

3. In the short run, some factors must be used in predetermined amounts. In the long run, all factors are free to vary.
4. If the firm is maximizing profits, then the value of the marginal product of each factor that it is free to vary must equal its factor price.
5. The logic of profit maximization implies that the supply function of a competitive firm must be an increasing function of the price of output and that each factor demand function must be a decreasing function of its price.
6. If a competitive firm exhibits constant returns to scale, then its long-run maximum profits must be zero.

REVIEW QUESTIONS

1. In the short run, if the price of the fixed factor is increased, what will happen to profits?
2. If a firm had everywhere increasing returns to scale, what would happen to its profits if prices remained fixed and if it doubled its scale of operation?
3. If a firm had decreasing returns to scale at all levels of output and it divided up into two equal-size smaller firms, what would happen to its overall profits?
4. A gardener exclaims: "For only \$1 in seeds I've grown over \$20 in produce!" Besides the fact that most of the produce is in the form of zucchini, what other observations would a cynical economist make about this situation?
5. Is maximizing a firm's profits always identical to maximizing the firm's stock market value?
6. If $pMP_1 > w_1$, then should the firm increase or decrease the amount of factor 1 in order to increase profits?
7. Suppose a firm is maximizing profits in the short run with variable factor x_1 and fixed factor x_2 . If the price of x_2 goes down, what happens to the firm's use of x_1 ? What happens to the firm's level of profits?
8. A profit-maximizing competitive firm that is making positive profits in long-run equilibrium (may/may not) have a technology with constant returns to scale.

APPENDIX

The profit-maximization problem of the firm is

$$\max_{x_1, x_2} pf(x_1, x_2) - w_1x_1 - w_2x_2,$$

which has first-order conditions

$$\begin{aligned} p \frac{\partial f(x_1^*, x_2^*)}{\partial x_1} - w_1 &= 0 \\ p \frac{\partial f(x_1^*, x_2^*)}{\partial x_2} - w_2 &= 0. \end{aligned}$$

These are just the same as the marginal product conditions given in the text. Let's see how profit-maximizing behavior looks using the Cobb-Douglas production function.

Suppose the Cobb-Douglas function is given by $f(x_1, x_2) = x_1^a x_2^b$. Then the two first-order conditions become

$$\begin{aligned} pax_1^{a-1}x_2^b - w_1 &= 0 \\ pbx_1^ax_2^{b-1} - w_2 &= 0. \end{aligned}$$

Multiply the first equation by x_1 and the second equation by x_2 to get

$$\begin{aligned} pax_1^ax_2^b - w_1x_1 &= 0 \\ pbx_1^ax_2^b - w_2x_2 &= 0. \end{aligned}$$

Using $y = x_1^ax_2^b$ to denote the level of output of this firm we can rewrite these expressions as

$$\begin{aligned} pay &= w_1x_1 \\ pby &= w_2x_2. \end{aligned}$$

Solving for x_1 and x_2 we have

$$\begin{aligned} x_1^* &= \frac{apy}{w_1} \\ x_2^* &= \frac{bpy}{w_2}. \end{aligned}$$

This gives us the demands for the two factors as a function of the optimal output choice. But we still have to solve for the optimal choice of output. Inserting the optimal factor demands into the Cobb-Douglas production function, we have the expression

$$\left(\frac{pay}{w_1}\right)^a \left(\frac{pby}{w_2}\right)^b = y.$$

Factoring out the y gives

$$\left(\frac{pa}{w_1}\right)^a \left(\frac{pb}{w_2}\right)^b y^{a+b} = y.$$

Or

$$y = \left(\frac{pa}{w_1} \right)^{\frac{a}{1-a-b}} \left(\frac{pb}{w_2} \right)^{\frac{b}{1-a-b}}.$$

This gives us the supply function of the Cobb-Douglas firm. Along with the factor demand functions derived above it gives us a complete solution to the profit-maximization problem.

Note that when the firm exhibits constant returns to scale—when $a + b = 1$ —this supply function is not well defined. As long as the output and input prices are consistent with zero profits, a firm with a Cobb-Douglas technology is indifferent about its level of supply.