CHAPTER 2

BUDGET CONSTRAINT

The economic theory of the consumer is very simple: economists assume that consumers choose the best bundle of goods they can afford. To give content to this theory, we have to describe more precisely what we mean by "best" and what we mean by "can afford." In this chapter we will examine how to describe what a consumer can afford; the next chapter will focus on the concept of how the consumer determines what is best. We will then be able to undertake a detailed study of the implications of this simple model of consumer behavior.

2.1 The Budget Constraint

We begin by examining the concept of the **budget constraint**. Suppose that there is some set of goods from which the consumer can choose. In real life there are many goods to consume, but for our purposes it is convenient to consider only the case of two goods, since we can then depict the consumer's choice behavior graphically.

We will indicate the consumer's **consumption bundle** by (x_1, x_2). This is simply a list of two numbers that tells us how much the consumer is choosing to consume of good 1, x_1, and how much the consumer is choosing to consume of good 2, x_2. Sometimes it is convenient to denote the consumer's bundle by a single symbol like X, where X is simply an abbreviation for the list of two numbers (x_1, x_2).

We suppose that we can observe the prices of the two goods, (p_1, p_2), and the amount of money the consumer has to spend, m. Then the budget constraint of the consumer can be written as

$$p_1x_1 + p_2x_2 \leq m. \tag{2.1}$$

Here p_1x_1 is the amount of money the consumer is spending on good 1, and p_2x_2 is the amount of money the consumer is spending on good 2. The budget constraint of the consumer requires that the amount of money spent on the two goods be no more than the total amount the consumer has to spend. The consumer's *affordable* consumption bundles are those that don't cost any more than m. We call this set of affordable consumption bundles at prices (p_1, p_2) and income m the **budget set** of the consumer.

2.2 Two Goods Are Often Enough

The two-good assumption is more general than you might think at first, since we can often interpret one of the goods as representing everything else the consumer might want to consume.

For example, if we are interested in studying a consumer's demand for milk, we might let x_1 measure his or her consumption of milk in quarts per month. We can then let x_2 stand for everything else the consumer might want to consume.

When we adopt this interpretation, it is convenient to think of good 2 as being the dollars that the consumer can use to spend on other goods. Under this interpretation the price of good 2 will automatically be 1, since the price of one dollar is one dollar. Thus the budget constraint will take the form

$$p_1x_1 + x_2 \leq m. \tag{2.2}$$

This expression simply says that the amount of money spent on good 1, p_1x_1, plus the amount of money spent on all other goods, x_2, must be no more than the total amount of money the consumer has to spend, m.

We say that good 2 represents a **composite good** that stands for everything else that the consumer might want to consume other than good 1. Such a composite good is invariably measured in dollars to be spent on goods other than good 1. As far as the algebraic form of the budget constraint is concerned, equation (2.2) is just a special case of the formula given in equation (2.1), with $p_2 = 1$, so everything that we have to say about the budget constraint in general will hold under the composite-good interpretation.

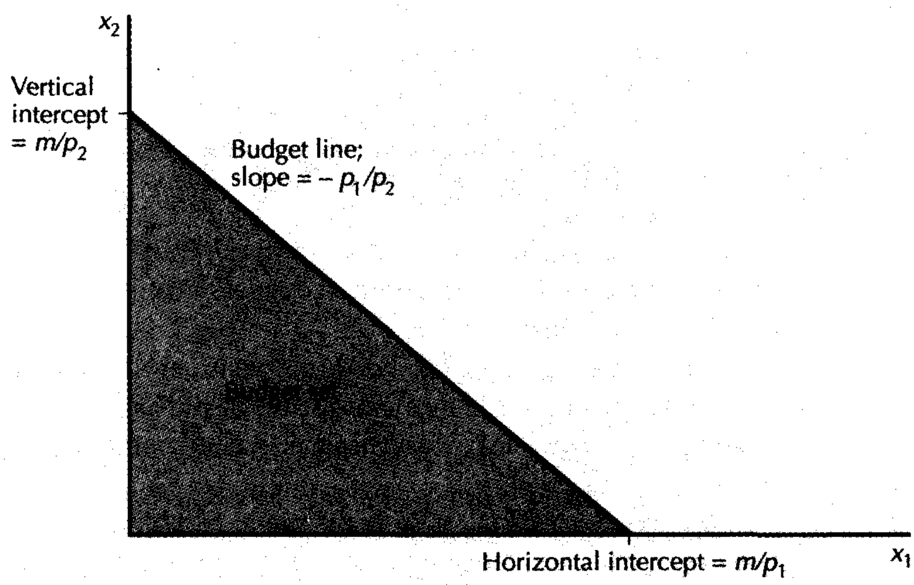
2.3 Properties of the Budget Set

The **budget line** is the set of bundles that cost exactly m:

$$p_1x_1 + p_2x_2 = m. \tag{2.3}$$

These are the bundles of goods that just exhaust the consumer's income.

The budget set is depicted in Figure 2.1. The heavy line is the budget line—the bundles that cost exactly m—and the bundles below this line are those that cost strictly less than m.



The budget set. The budget set consists of all bundles that are affordable at the given prices and income.

We can rearrange the budget line in equation (2.3) to give us the formula

$$x_2 = \frac{m}{p_2} - \frac{p_1}{p_2}x_1. \tag{2.4}$$

This is the formula for a straight line with a vertical intercept of m/p_2 and a slope of $-p_1/p_2$. The formula tells us how many units of good 2 the consumer needs to consume in order to just satisfy the budget constraint if she is consuming x_1 units of good 1.

Here is an easy way to draw a budget line given prices (p_1, p_2) and income m. Just ask yourself how much of good 2 the consumer could buy if she spent all of her money on good 2. The answer is, of course, m/p_2. Then ask how much of good 1 the consumer could buy if she spent all of her money on good 1. The answer is m/p_1. Thus the horizontal and vertical intercepts measure how much the consumer could get if she spent all of her money on goods 1 and 2, respectively. In order to depict the budget line just plot these two points on the appropriate axes of the graph and connect them with a straight line.

The slope of the budget line has a nice economic interpretation. It measures the rate at which the market is willing to "substitute" good 1 for good 2. Suppose for example that the consumer is going to increase her consumption of good 1 by Δx_1.[1] How much will her consumption of good 2 have to change in order to satisfy her budget constraint? Let us use Δx_2 to indicate her change in the consumption of good 2.

Now note that if she satisfies her budget constraint before and after making the change she must satisfy

$$p_1 x_1 + p_2 x_2 = m$$

and

$$p_1(x_1 + \Delta x_1) + p_2(x_2 + \Delta x_2) = m.$$

Subtracting the first equation from the second gives

$$p_1 \Delta x_1 + p_2 \Delta x_2 = 0.$$

This says that the total value of the change in her consumption must be zero. Solving for $\Delta x_2/\Delta x_1$, the rate at which good 2 can be substituted for good 1 while still satisfying the budget constraint, gives

$$\frac{\Delta x_2}{\Delta x_1} = -\frac{p_1}{p_2}.$$

This is just the slope of the budget line. The negative sign is there since Δx_1 and Δx_2 must always have opposite signs. If you consume more of good 1, you have to consume less of good 2 and vice versa if you continue to satisfy the budget constraint.

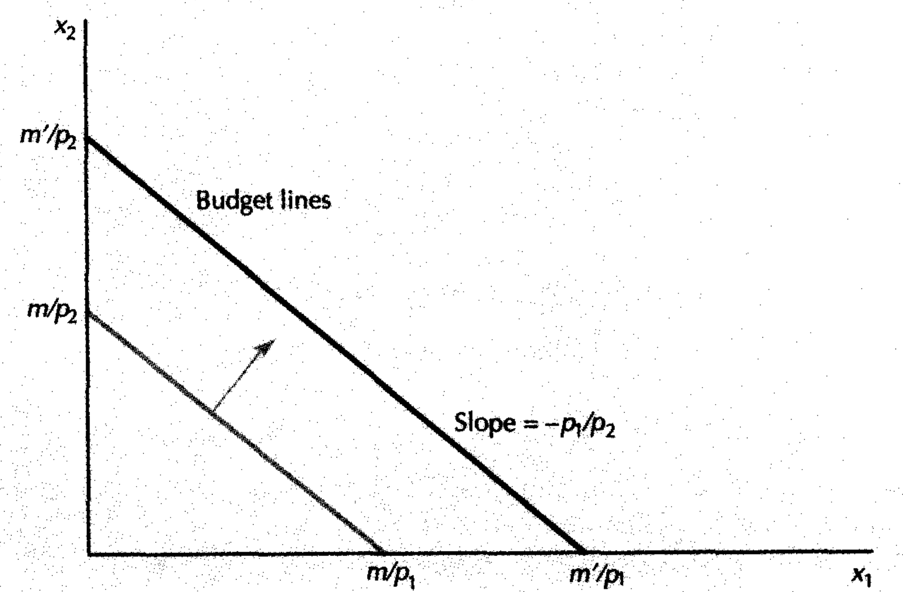
Economists sometimes say that the slope of the budget line measures the **opportunity cost** of consuming good 1. In order to consume more of good 1 you have to give up some consumption of good 2. Giving up the opportunity to consume good 2 is the true economic cost of more good 1 consumption; and that cost is measured by the slope of the budget line.

[1] The Greek letter Δ, delta, is pronounced "del-ta." The notation Δx_1 denotes the change in good 1. For more on changes and rates of changes, see the Mathematical Appendix.

2.4 How the Budget Line Changes

When prices and incomes change, the set of goods that a consumer can afford changes as well. How do these changes affect the budget set?

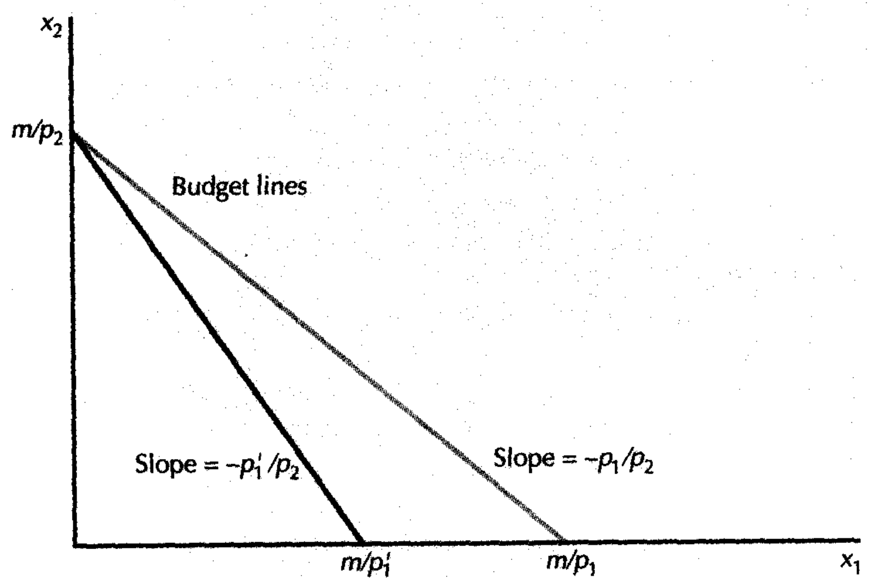
Let us first consider changes in income. It is easy to see from equation (2.4) that an increase in income will increase the vertical intercept and not affect the slope of the line. Thus an increase in income will result in a *parallel shift outward* of the budget line as in Figure 2.2. Similarly, a decrease in income will cause a parallel shift inward.



Increasing income. An increase in income causes a parallel shift outward of the budget line.

What about changes in prices? Let us first consider increasing price 1 while holding price 2 and income fixed. According to equation (2.4), increasing p_1 will not change the vertical intercept, but it will make the budget line steeper since p_1/p_2 will become larger.

Another way to see how the budget line changes is to use the trick described earlier for drawing the budget line. If you are spending all of your money on good 2, then increasing the price of good 1 doesn't change the maximum amount of good 2 you could buy—thus the vertical intercept of the budget line doesn't change. But if you are spending all of your money on good 1, and good 1 becomes more expensive, then your consumption of good 1 must decrease. Thus the horizontal intercept of the budget line must shift inward, resulting in the tilt depicted in Figure 2.3.



Increasing price. **If good 1 becomes more expensive, the budget line becomes steeper.**

What happens to the budget line when we change the prices of good 1 and good 2 at the same time? Suppose for example that we double the prices of both goods 1 and 2. In this case both the horizontal and vertical intercepts shift inward by a factor of one-half, and therefore the budget line shifts inward by one-half as well. Multiplying both prices by two is just like dividing income by 2.

We can also see this algebraically. Suppose our original budget line is

$$p_1 x_1 + p_2 x_2 = m.$$

Now suppose that both prices become t times as large. Multiplying both prices by t yields

$$tp_1 x_1 + tp_2 x_2 = m.$$

But this equation is the same as

$$p_1 x_1 + p_2 x_2 = \frac{m}{t}.$$

Thus multiplying both prices by a constant amount t is just like dividing income by the same constant t. It follows that if we multiply both prices by t *and* we multiply income by t, then the budget line won't change at all.

We can also consider price and income changes together. What happens if both prices go up and income goes down? Think about what happens to the horizontal and vertical intercepts. If m decreases and p_1 and p_2 both increase, then the intercepts m/p_1 and m/p_2 must both decrease. This means that the budget line will shift inward. What about the slope of the budget line? If price 2 increases more than price 1, so that $-p_1/p_2$ decreases (in absolute value), then the budget line will be flatter; if price 2 increases less than price 1, the budget line will be steeper.

2.5 The Numeraire

The budget line is defined by two prices and one income, but one of these variables is redundant. We could peg one of the prices, or the income, to some fixed value, and adjust the other variables so as to describe exactly the same budget set. Thus the budget line

$$p_1x_1 + p_2x_2 = m$$

is exactly the same budget line as

$$\frac{p_1}{p_2}x_1 + x_2 = \frac{m}{p_2}$$

or

$$\frac{p_1}{m}x_1 + \frac{p_2}{m}x_2 = 1,$$

since the first budget line results from dividing everything by p_2, and the second budget line results from dividing everything by m. In the first case, we have pegged $p_2 = 1$, and in the second case, we have pegged $m = 1$. Pegging the price of one of the goods or income to 1 and adjusting the other price and income appropriately doesn't change the budget set at all.

When we set one of the prices to 1, as we did above, we often refer to that price as the **numeraire** price. The numeraire price is the price relative to which we are measuring the other price and income. It will occasionally be convenient to think of one of the goods as being a numeraire good, since there will then be one less price to worry about.

2.6 Taxes, Subsidies, and Rationing

Economic policy often uses tools that affect a consumer's budget constraint, such as taxes. For example, if the government imposes a **quantity tax**, this means that the consumer has to pay a certain amount to the government for each unit of the good he purchases. In the U.S., for example, we pay about 15 cents a gallon as a federal gasoline tax.

How does a quantity tax affect the budget line of a consumer? From the viewpoint of the consumer the tax is just like a higher price. Thus a quantity tax of t dollars per unit of good 1 simply changes the price of good 1 from p_1 to $p_1 + t$. As we've seen above, this implies that the budget line must get steeper.

Another kind of tax is a **value** tax. As the name implies this is a tax on the value—the price—of a good, rather than the quantity purchased of a good. A value tax is usually expressed in percentage terms. Most states in the U.S. have sales taxes. If the sales tax is 6 percent, then a good that is priced at \$1 will actually sell for \$1.06. (Value taxes are also known as **ad valorem** taxes.)

If good 1 has a price of p_1 but is subject to a sales tax at rate τ, then the actual price facing the consumer is $(1 + \tau)p_1$.[2] The consumer has to pay p_1 to the supplier and τp_1 to the government for each unit of the good so the total cost of the good to the consumer is $(1 + \tau)p_1$.

A **subsidy** is the opposite of a tax. In the case of a **quantity subsidy**, the government *gives* an amount to the consumer that depends on the amount of the good purchased. If, for example, the consumption of milk were subsidized, the government would pay some amount of money to each consumer of milk depending on the amount that consumer purchased. If the subsidy is s dollars per unit of consumption of good 1, then from the viewpoint of the consumer, the price of good 1 would be $p_1 - s$. This would therefore make the budget line flatter.

Similarly an ad valorem subsidy is a subsidy based on the price of the good being subsidized. If the government gives you back \$1 for every \$2 you donate to charity, then your donations to charity are being subsidized at a rate of 50 percent. In general, if the price of good 1 is p_1 and good 1 is subject to an ad valorem subsidy at rate σ, then the actual price of good 1 facing the consumer is $(1 - \sigma)p_1$.[3]

You can see that taxes and subsidies affect prices in exactly the same way except for the algebraic sign: a tax increases the price to the consumer, and a subsidy decreases it.

Another kind of tax or subsidy that the government might use is a **lump-sum** tax or subsidy. In the case of a tax, this means that the government takes away some fixed amount of money, regardless of the individual's behavior. Thus a lump-sum tax means that the budget line of a consumer will shift inward because his money income has been reduced. Similarly, a lump-sum subsidy means that the budget line will shift outward. Quantity taxes and value taxes tilt the budget line one way or the other depending

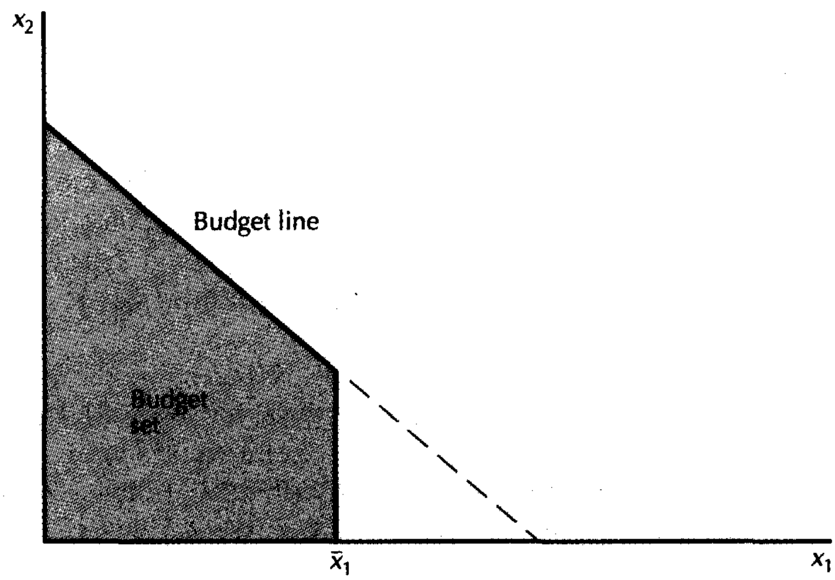
[2] The Greek letter τ, tau, rhymes with "wow."

[3] The Greek letter σ is pronounced "sig-ma."

on which good is being taxed, but a lump-sum tax shifts the budget line inward.

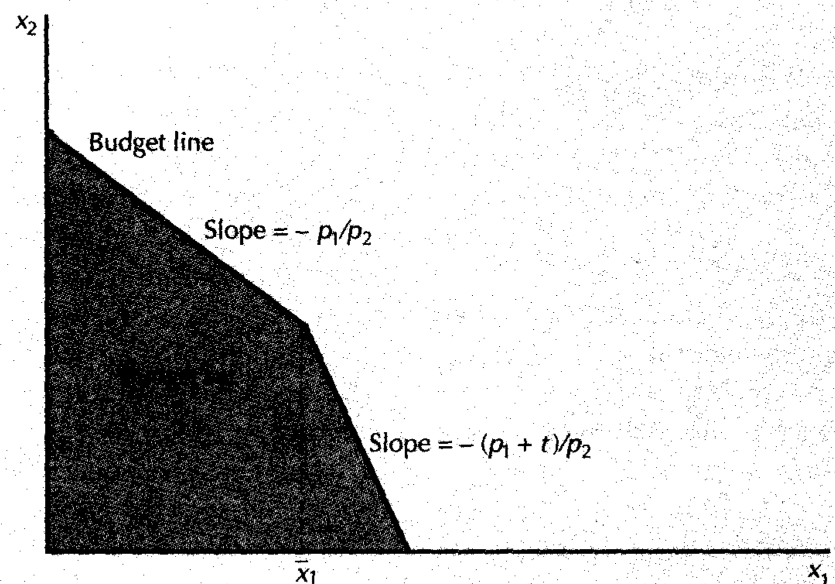
Governments also sometimes impose *rationing* constraints. This means that the level of consumption of some good is fixed to be no larger than some amount. For example, during World War II the U.S. government rationed certain foods like butter and meat.

Suppose, for example, that good 1 were rationed so that no more than $\overline{x}_1$ could be consumed by a given consumer. Then the budget set of the consumer would look like that depicted in Figure 2.4: it would be the old budget set with a piece lopped off. The lopped-off piece consists of all the consumption bundles that are affordable but have $x_1 > \overline{x}_1$.



Budget set with rationing. If good 1 is rationed, the section of the budget set beyond the rationed quantity will be lopped off.

Sometimes taxes, subsidies, and rationing are combined. For example, we could consider a situation where a consumer could consume good 1 at a price of p_1 up to some level $\overline{x}_1$, and then had to pay a tax t on all consumption in excess of $\overline{x}_1$. The budget set for this consumer is depicted in Figure 2.5. Here the budget line has a slope of $-p_1/p_2$ to the left of $\overline{x}_1$, and a slope of $-(p_1 + t)/p_2$ to the right of $\overline{x}_1$.



Taxing consumption greater than $\bar{x}_1$. In this budget set the consumer must pay a tax only on the consumption of good 1 that is in excess of $\bar{x}_1$, so the budget line becomes steeper to the right of $\bar{x}_1$.

EXAMPLE: The Food Stamp Program

Since the Food Stamp Act of 1964 the U.S. federal government has provided a subsidy on food for poor people. The details of this program have been adjusted several times. Here we will describe the economic effects of one of these adjustments.

Before 1979, households who met certain eligibility requirements were allowed to purchase food stamps, which could then be used to purchase food at retail outlets. In January 1975, for example, a family of four could receive a maximum monthly allotment of $153 in food coupons by participating in the program.

The price of these coupons to the household depended on the household income. A family of four with an adjusted monthly income of $300 paid $83 for the full monthly allotment of food stamps. If a family of four had a monthly income of $100, the cost for the full monthly allotment would have been $25.[4]

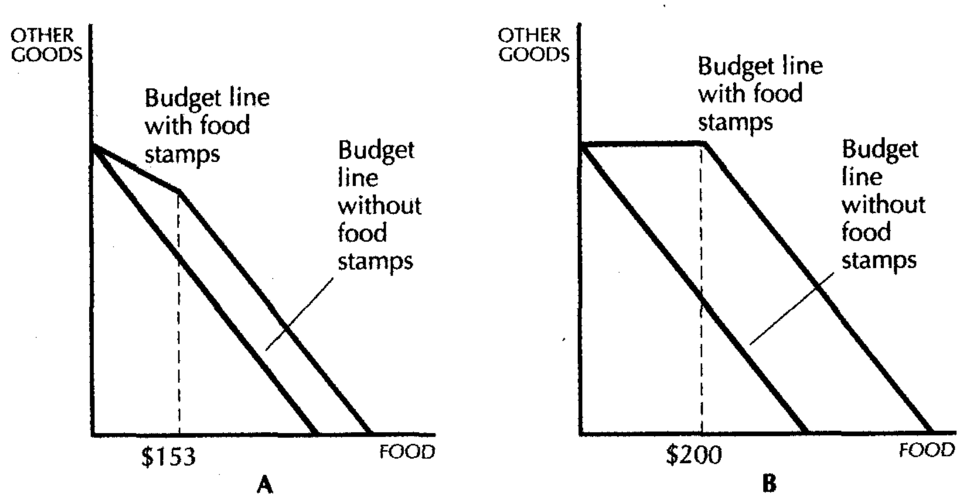
The pre-1979 Food Stamp program was an ad valorem subsidy on food. The rate at which food was subsidized depended on the household income.

[4] These figures are taken from Kenneth Clarkson, *Food Stamps and Nutrition*, American Enterprise Institute, 1975.

The family of four that was charged \$83 for their allotment paid \$1 to receive \$1.84 worth of food (1.84 equals 153 divided by 83). Similarly, the household that paid \$25 was paying \$1 to receive \$6.12 worth of food (6.12 equals 153 divided by 25).

The way that the Food Stamp program affected the budget set of a household is depicted in Figure 2.6A. Here we have measured the amount of money spent on food on the horizontal axis and expenditures on all other goods on the vertical axis. Since we are measuring each good in terms of the money spent on it, the "price" of each good is automatically 1, and the budget line will therefore have a slope of -1.

If the household is allowed to buy \$153 of food stamps for \$25, then this represents roughly an 84 percent ($= 1 - 25/153$) subsidy of food purchases, so the budget line will have a slope of roughly $-.16$ ($= 25/153$) until the household has spent \$153 on food. Each dollar that the household spends on food up to \$153 would reduce its consumption of other goods by about 16 cents. After the household spends \$153 on food, the budget line facing it would again have a slope of -1.



Food stamps. How the budget line is affected by the Food Stamp program. Part A shows the pre-1979 program and part B the post-1979 program.

These effects lead to the kind of "kink" depicted in Figure 2.6. Households with higher incomes had to pay more for their allotment of food stamps. Thus the slope of the budget line would become steeper as household income increased.

In 1979 the Food Stamp program was modified. Instead of requiring that households purchase food stamps, they are now simply given to qualified households. Figure 2.6B shows how this affects the budget set.

Suppose that a household now receives a grant of \$200 of food stamps a month. Then this means that the household can consume \$200 more food per month, regardless of how much it is spending on other goods, which implies that the budget line will shift to the right by \$200. The slope will not change: \$1 less spent on food would mean \$1 more to spend on other things. But since the household cannot legally sell food stamps, the maximum amount that it can spend on other goods does not change. The Food Stamp program is effectively a lump-sum subsidy, except for the fact that the food stamps can't be sold.

2.7 Budget Line Changes

In the next chapter we will analyze how the consumer chooses an optimal consumption bundle from his or her budget set. But we can already state some observations here that follow from what we have learned about the movements of the budget line.

First, we can observe that since the budget set doesn't change when we multiply all prices and income by a positive number, the optimal choice of the consumer from the budget set can't change either. Without even analyzing the choice process itself, we have derived an important conclusion: a perfectly balanced inflation—one in which all prices and all incomes rise at the same rate—doesn't change anybody's budget set, and thus cannot change anybody's optimal choice.

Second, we can make some statements about how well-off the consumer can be at different prices and incomes. Suppose that the consumer's income increases and all prices remain the same. We know that this represents a parallel shift outward of the budget line. Thus every bundle the consumer was consuming at the lower income is also a possible choice at the higher income. But then the consumer must be at least as well-off at the higher income as at the lower income—since he or she has the same choices available as before plus some more. Similarly, if one price declines and all others stay the same, the consumer must be at least as well-off. This simple observation will be of considerable use later on.

Summary

1. The budget set consists of all bundles of goods that the consumer can afford at given prices and income. We will typically assume that there are only two goods, but this assumption is more general than it seems.

2. The budget line is written as $p_1x_1 + p_2x_2 = m$. It has a slope of $-p_1/p_2$, a vertical intercept of m/p_2, and a horizontal intercept of m/p_1.

3. Increasing income shifts the budget line outward. Increasing the price of good 1 makes the budget line steeper. Increasing the price of good 2 makes the budget line flatter.

4. Taxes, subsidies, and rationing change the slope and position of the budget line by changing the prices paid by the consumer.

REVIEW QUESTIONS

1. Originally the consumer faces the budget line $p_1x_1 + p_2x_2 = m$. Then the price of good 1 doubles, the price of good 2 becomes 8 times larger, and income becomes 4 times larger. Write down an equation for the new budget line in terms of the original prices and income.

2. What happens to the budget line if the price of good 2 increases, but the price of good 1 and income remain constant?

3. If the price of good 1 doubles and the price of good 2 triples, does the budget line become flatter or steeper?

4. What is the definition of a numeraire good?

5. Suppose that the government puts a tax of 15 cents a gallon on gasoline and then later decides to put a subsidy on gasoline at a rate of 7 cents a gallon. What net tax is this combination equivalent to?

6. Suppose that a budget equation is given by $p_1x_1 + p_2x_2 = m$. The government decides to impose a lump-sum tax of u, a quantity tax on good 1 of t, and a quantity subsidy on good 2 of s. What is the formula for the new budget line?

7. If the income of the consumer increases and one of the prices decreases at the same time, will the consumer necessarily be at least as well-off?