CHAPTER 32

PRODUCTION

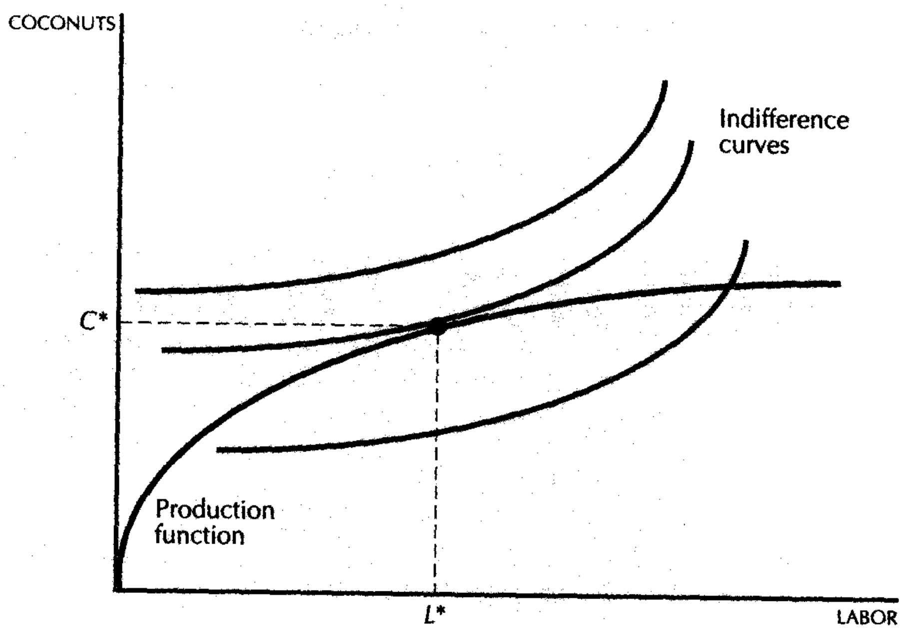
In the last chapter we described a general equilibrium model of a pure exchange economy and discussed issues of resource allocation when a fixed amount of each good was available. In this chapter we want to describe how production fits into the general equilibrium framework. When production is possible, the amounts of the goods are not fixed but will respond to market prices.

If you thought the two-consumer two-good assumption was a restrictive framework in which to examine trade, imagine what production is going to look like! The minimal set of players that we can have to make an interesting problem is one consumer, one firm, and two goods. The traditional name for this economic model is the **Robinson Crusoe economy**, after Defoe's shipwrecked hero.

32.1 The Robinson Crusoe Economy

In this economy Robinson Crusoe plays a dual role: he is both a consumer and a producer. Robinson can spend his time loafing on the beach thereby consuming leisure, or he can spend time gathering coconuts. The more coconuts he gathers the more he has to eat, but the less time he has to improve his tan.

Robinson's preferences for coconuts and leisure are depicted in Figure 32.1. They are just like the preferences for leisure and consumption depicted in Chapter 9, except we are measuring labor on the horizontal axis rather than leisure. So far nothing new has been added.



The Robinson Crusoe economy. The indifference curves depict Robinson's preferences for coconuts and leisure. The production function depicts the technological relationship between the amount he works and the amount of coconuts he produces.

Now let's draw in the **production function**, the function that illustrates the relationship between how much Robinson works and how many coconuts he gets. This will typically have the shape depicted in Figure 32.1. The more Robinson works, the more coconuts he will get; but, due to diminishing returns to labor, the marginal product of his labor declines: the number of extra coconuts that he gets from an extra hour's labor decreases as the hours of labor increase.

How much does Robinson work and how much does he consume? To answer these questions, look for the highest indifference curve that just touches the production set. This will give the most-preferred combination of labor and consumption that Robinson can get, given the technology for gathering coconuts that he is using.

At this point, the slope of the indifference curve must equal the slope of the production function by the standard argument: if they crossed, there would be some other feasible point that was preferred. This means that the marginal product of an extra hour of labor must equal the marginal rate of substitution between leisure and coconuts. If the marginal product were greater than the marginal rate of substitution, it would pay for Robinson to give up a little leisure in order to get the extra coconuts. If the marginal product were less than the marginal rate of substitution, it would pay for Robinson to work a little less.

32.2 Crusoe, Inc.

So far this story is only a slight extension of models we have already seen. But now let's add a new feature. Suppose that Robinson is tired of simultaneously being a producer and consumer and that he decides to alternate roles. One day he will behave entirely as a producer, and the next day he will behave entirely as a consumer. In order to coordinate these activities, he decides to set up a labor market and a coconut market.

He also sets up a firm, Crusoe, Inc., and becomes its sole shareholder. The firm is going to look at the prices for labor and coconuts and decide how much labor to hire and how many coconuts to produce, guided by the principle of profit maximization. Robinson, in his role as a worker, is going to collect income from working at the firm; in his role as shareholder in the firm he will collect profits; and, in his role as consumer he will decide how much to purchase of the firm's output. (No doubt this sounds peculiar, but there really isn't that much else to do on a desert island.)

In order to keep track of his transactions, Robinson invents a currency he calls "dollars," and he chooses, somewhat arbitrarily, to set the price of coconuts at one dollar apiece. Thus coconuts are the numeraire good for this economy; as we've seen in Chapter 2, a numeraire good is one whose price has been set to one. Since the price of coconuts is normalized at one, we have only to determine the wage rate. What should his wage rate be in order to make this market work?

We're going to think about this problem first from the viewpoint of Crusoe, Inc., and then from the viewpoint of Robinson, the consumer. The discussion is a little schizophrenic at times, but that's what you have to put up with if you want to have an economy with only one person. We're going to look at the economy after it has been running along for some time, and everything is in equilibrium. In equilibrium, the demand for coconuts will equal the supply of coconuts and the demand for labor will equal the supply of labor. Both Crusoe, Inc. and Robinson the consumer will be making optimal choices given the constraints they face.

32.3 The Firm

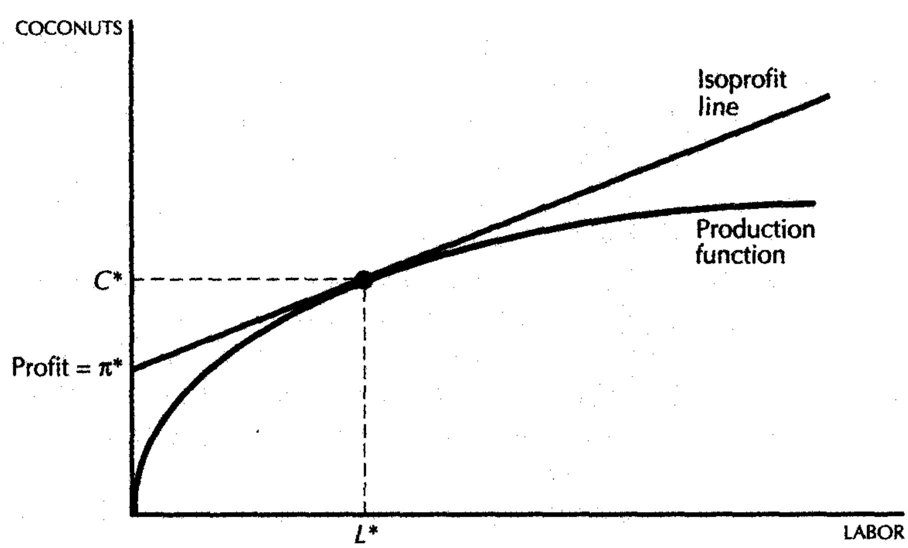
Each evening, Crusoe, Inc. decides how much labor it wants to hire the next day, and how many coconuts it wants to produce. Given a price of coconuts of 1 and a wage rate of labor of w, we can solve the firm's profit-maximization problem in Figure 32.2. We first consider all combinations of coconuts and labor that yield a constant level of profits, π. This means that

$$\pi = C - wL.$$

Solving for C, we have

$$C = \pi + wL.$$

Just as in Chapter 19, this formula describes the isoprofit lines—all combinations of labor and coconuts that yield profits of π. Crusoe, Inc. will choose a point where the profits are maximized. As usual, this implies a tangency condition: the slope of the production function—the marginal product of labor—must equal w, as illustrated in Figure 32.2.



Profit maximization. Crusoe, Inc. chooses a production plan that maximizes profits. At the optimal point the production function must be tangent to an isoprofit line.

Thus the vertical intercept of the isoprofit line measures the maximal level of profits measured in units of coconuts: if Robinson generates π^* dollars of profit, this money can buy π^* coconuts, since the price of coconuts has been chosen to be 1. There we have it. Crusoe, Inc. has done its job. Given the wage w, it has determined how much labor it wants to hire, how many coconuts it wants to produce, and what profits it will generate by following this plan. So Crusoe, Inc. declares a stock dividend of π^* dollars and mails it off to its sole shareholder, Robinson.

32.4 Robinson's Problem

The next day Robinson wakes up and receives his dividend of π^* dollars. While eating his coconut breakfast, he contemplates how much he wants to work and consume. He may consider just consuming his endowment—spend his profits on π^* coconuts and consume his endowment of leisure. But listening to his stomach growl is not so pleasant, and it might make sense to work for a few hours instead. So Robinson trudges down to Crusoe, Inc. and starts to gather coconuts, just as he has done every other day.

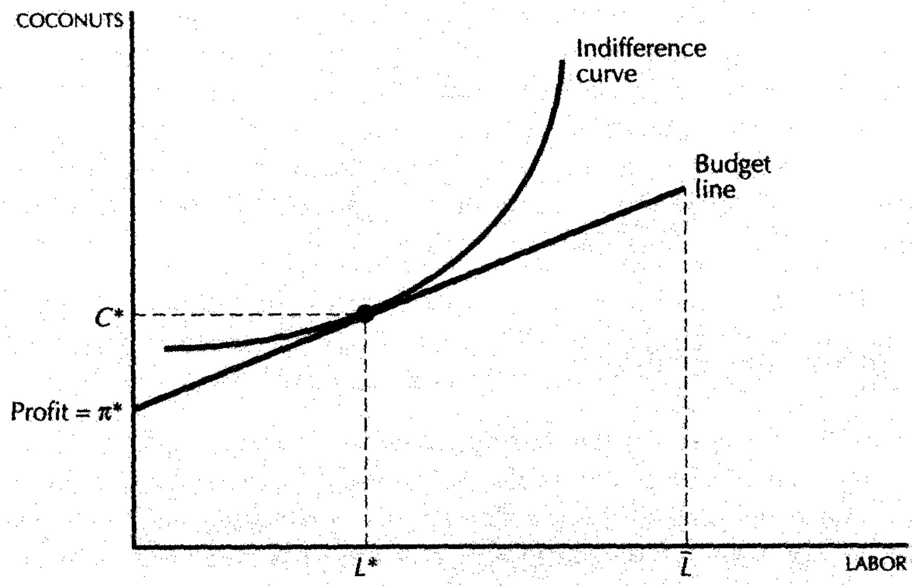
We can describe Robinson's labor-consumption choice using standard indifference curve analysis. Plotting labor on the horizontal axis and coconuts on the vertical axis, we can draw in an indifference curve as illustrated in Figure 32.3.

Since labor is a bad, by assumption, and coconuts are a good, the indifference curve has a positive slope as shown in the diagram. If we indicate the maximum amount of labor by $\overline{L}$, then the distance from $\overline{L}$ to the chosen supply of labor gives Robinson's demand for leisure. This is just like the supply of labor model examined in Chapter 9, except we have reversed the origin on the horizontal axis.

Robinson's budget line is also illustrated in Figure 32.3. It has a slope of w and passes through his endowment point $(\pi^*, 0)$. (Robinson has a zero endowment of labor and a π^* endowment of coconuts since that would be his bundle if he engaged in no market transactions.) Given the wage rate, Robinson chooses optimally how much he wants to work and how many coconuts he wants to consume. At his optimal consumption, the marginal rate of substitution between consumption and leisure must equal the wage rate, just as in a standard consumer choice problem.

32.5 Putting Them Together

Now we superimpose Figures 32.2 and 32.3 to get Figure 32.4. Look at what has happened! Robinson's bizarre behavior has worked out all right after all. He ends up consuming at exactly the same point as he would have if he had made all the decisions at once. Using the market system



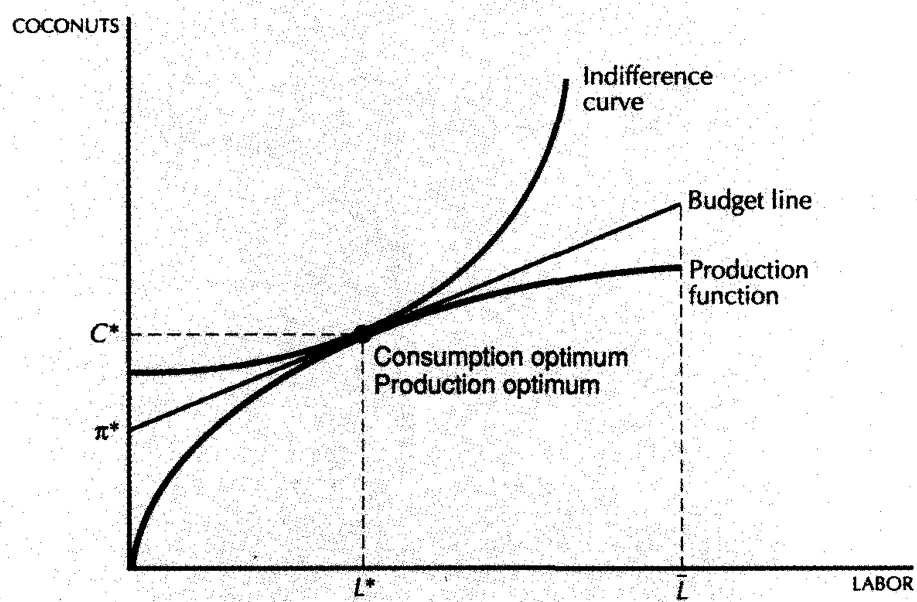
Robinson's maximization problem. Robinson the consumer decides how much to work and consume given the prices and wages. The optimal point will occur where the indifference curve is tangent to the budget line.

results in the same outcome as choosing the consumption and production plans directly.

Since the marginal rate of substitution between leisure and consumption equals the wage, and the marginal product of labor equals the wage, we are assured that the marginal rate of substitution between labor and consumption equals the marginal product—that is, that the slopes of the indifference curve and the production set are the same.

In the case of a one-person economy, using the market is pretty silly. Why should Robinson bother to break up his decision into two pieces? But in an economy with many people, breaking up decisions no longer seems so odd. If there are many firms, then questioning each person about how much they want of each good is simply impractical. In a market economy the firms simply have to look at the prices of goods in order to make their production decisions. For the prices of goods measure how much the consumers value *extra* units of consumption. And the decision that the firms face, for the most part, is whether they should produce more or less output.

The market prices reflect the marginal values of the goods that the firms use as inputs and outputs. If firms use the change in profits as a guide to production, where the profits are measured at market prices, then their decisions will reflect the marginal values that consumers place on the goods.



Equilibrium in both consumption and production. The amount of coconuts demanded by the consumer Robinson equals the amount of coconuts supplied by Crusoe, Inc.

32.6 Different Technologies

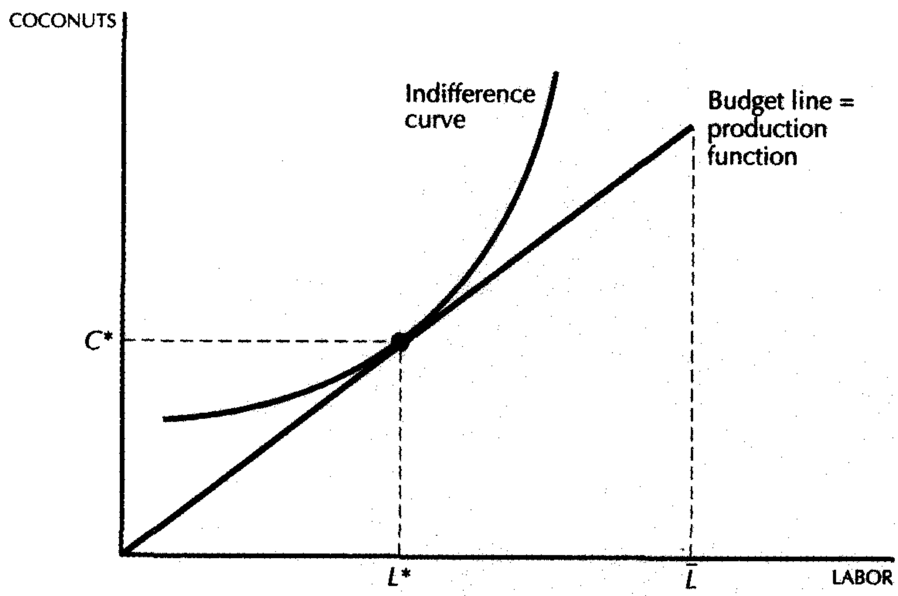
In the above discussion we have assumed that the technology available to Robinson exhibited diminishing returns to labor. Since labor was the only input to production, this was equivalent to decreasing returns to scale. (This is not necessarily true if there is more than one input!)

It is useful to consider some other possibilities. Suppose, for example, that the technology exhibited constant returns to scale. Recall that constant returns to scale means that using twice as much of all inputs produces twice as much output. In the case of a one-input production function, this means that the production function must be a straight line through the origin as depicted in Figure 32.5.

Since the technology has constant returns to scale, the argument in Chapter 19 implies that the only reasonable operating position for a competitive firm is at zero profits. This is because if the profits were ever greater than zero, it would pay for the firm to expand output indefinitely, and if profits were ever less than zero, it would pay the firm to produce zero output.

Thus Robinson's endowment involves zero profits and $\overline{L}$, his initial endowment of labor time. His budget set coincides with the production set, and the story is much the same as before.

The situation is somewhat different with an increasing returns to scale



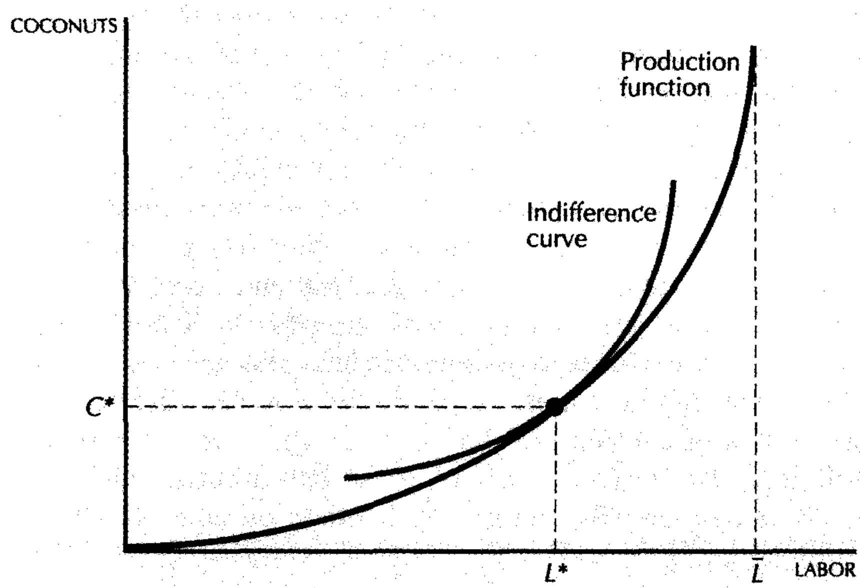
Constant returns to scale. If the technology exhibits constant returns to scale, Crusoe, Inc., makes zero profits.

technology, as depicted in Figure 32.6. There is no difficulty in this simple example in exhibiting the optimal choice of consumption and leisure for Robinson. The indifference curve will be tangent to the production set as usual. The problem arises in trying to support this point as a profit-maximizing point. For if the firm were faced with the prices given by Robinson's marginal rate of substitution, it would want to produce more output than Robinson would demand.

If the firm exhibits increasing returns to scale at the optimal choice, then the average costs of production will exceed the marginal costs of production—and that means that the firm will be making negative profits. The goal of profit maximization would lead the firm to want to increase its output—but this would be incompatible with the demands for output and the supplies of inputs from the consumers. In the case depicted, there is *no* price at which the utility-maximizing demand by the consumer will equal the profit-maximizing supply from the firm.

Increasing returns to scale is an example of a **nonconvexity**. In this case the production set—the set of coconuts and labor that are feasible for the economy—is not a convex set. Thus the common tangent to the indifference curve and the production function at the point (L^*, C^*) in Figure 32.6 will not separate the preferred points from the feasible points as it does in Figure 32.4.

Nonconvexities such as this pose grave difficulties for the functioning of competitive markets. In a competitive market consumers and firms look



Increasing returns to scale. The production set exhibits increasing returns to scale and the Pareto efficient allocation cannot be achieved by a competitive market.

at just one set of numbers—the market prices—to determine their consumption and production decisions. If the technology and the preferences are convex, then the only things that the economic agents need to know to make efficient decisions are the relationship between the prices and the marginal rates of substitution near the points where the economy is currently producing: the prices tell the agents everything that is necessary in order to determine an efficient allocation of resources.

But if the technology and/or the preferences are nonconvex, then the prices do not convey all the information necessary in order to choose an efficient allocation. Information about the slopes of the production function and indifference curves far away from the current operating position is also necessary.

However, these observations apply only when the returns to scale are large relative to the size of the market. Small regions of increasing returns to scale do not pose undue difficulties for a competitive market.

32.7 Production and the First Welfare Theorem

Recall that in the case of a pure exchange economy, a competitive equilibrium is Pareto efficient. This fact is known as the First Theorem of Welfare Economics. Does the same result hold in an economy with production? The diagrammatic approach used above is not adequate to answer this question, but a generalization of the algebraic argument we provided in Chapter 31 does nicely. It turns out that the answer is yes: if all firms act as competitive profit maximizers, then a competitive equilibrium will be Pareto efficient.

This result has the usual caveats. First, it has nothing to do with distribution. Profit maximization only ensures efficiency, not justice! Second, this result only makes sense when a competitive equilibrium actually exists. In particular, this will rule out large areas of increasing returns to scale. Third, the theorem implicitly assumes that the choices of any one firm do not affect the production possibilities of other firms. That is, it rules out the possibility of **production externalities**. Similarly, the theorem requires that firms' production decisions do not directly affect the consumption possibilities of consumers; that is, that there are no **consumption externalities**. More precise definitions of externalities will be given in Chapter 33, where we will examine their effect on efficient allocations in more detail.

32.8 Production and the Second Welfare Theorem

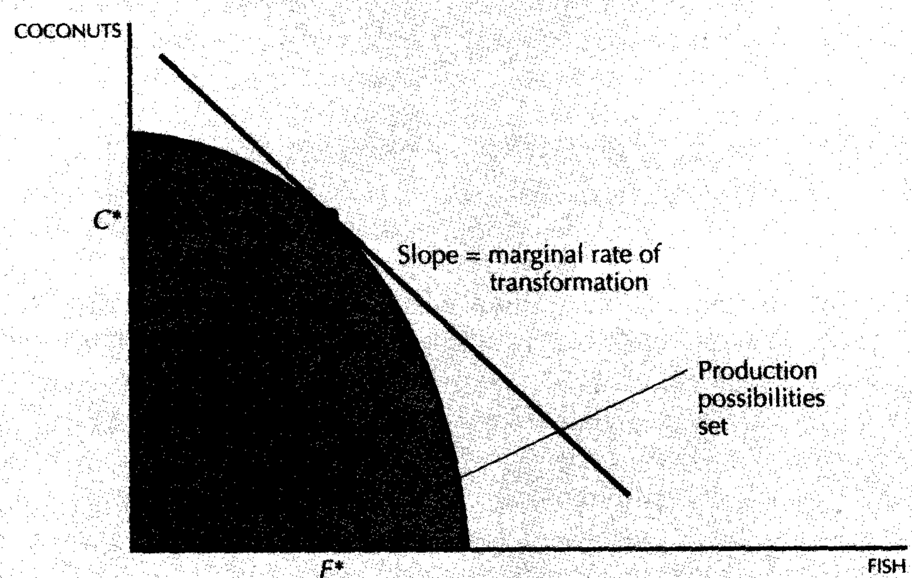
In the case of a pure exchange economy, every Pareto efficient allocation is a possible competitive equilibrium, as long as consumers exhibit convex preferences. In the case of an economy involving production, the same result is true, but now we require not only that consumers' preferences are convex, but also that firms' production sets are convex. As discussed above, this requirement effectively rules out the possibility of increasing returns to scale: if firms have increasing returns to scale at the equilibrium level of production, they would want to produce more output at the competitive prices.

However, with constant or decreasing returns to scale, the Second Welfare Theorem works fine. Any Pareto efficient allocation can be achieved through the use of competitive markets. Of course in general it will be necessary to redistribute endowments among the consumers to support different Pareto efficient allocations. In particular, both the income from endowments of labor and ownership shares of the firm will have to be redistributed. As indicated in the last chapter, there may be significant practical difficulties involved with this sort of redistribution.

32.9 Production Possibilities

We have now seen how production and consumption decisions can be made in a one-input, one-output economy. In this section we want to explore how this model can be generalized to an economy with several inputs and outputs. Although we will deal only with the two-good case, the concepts will generalize naturally to many goods.

So let us suppose that there is some other good that Robinson might produce—say fish. He can devote his time to gathering coconuts or to fishing. In Figure 32.7 we have depicted the various combinations of coconuts and fish that Robinson can produce from devoting different amounts of time to each activity. This set is known as a **production possibilities set**. The boundary of the production possibilities set is called the **production possibilities frontier**. This should be contrasted with the production function discussed earlier that depicts the relationship between the input good and the output good; the production possibilities set depicts only the set of *output* goods that is feasible. (In more advanced treatments, both inputs and outputs can be considered part of the production possibilities set, but these treatments cannot easily be handled with two-dimensional diagrams.)



A production possibilities set. The production possibilities set measures the set of outputs that are feasible given the technology and the amounts of inputs.

The shape of the production possibilities set will depend on the nature of the underlying technologies. If the technologies for producing coconuts and fish exhibit constant returns to scale the production possibilities set will take an especially simple form. Since by assumption there is only one input to production—Robinson's labor—the production functions for fish and coconuts will be simply *linear* functions of labor.

For example, suppose that Robinson can produce 10 pounds of fish per hour or 20 pounds of coconuts per hour. Then if he devotes L_f hours to fish production and L_c hours to coconut production, he will produce $10L_f$ pounds of fish and $20L_c$ pounds of coconuts. Suppose that Robinson decides to work 10 hours a day. Then the production possibilities set will consist of all combinations of coconuts, C, and fish, F, such that

$$\begin{aligned} F &= 10L_f \\ C &= 20L_c \\ L_c + L_f &= 10. \end{aligned}$$

The first two equations measure the production relationships, and the third measures the resource constraint. To determine the production possibilities frontier solve the first two equations for L_f and L_c to get

$$\begin{aligned} L_f &= \frac{F}{10} \\ L_c &= \frac{C}{20}. \end{aligned}$$

Now add these two equations together, and use the fact that $L_f + L_c = 10$ to find

$$\frac{F}{10} + \frac{C}{20} = 10.$$

This equation gives us all the combinations of fish and coconuts that Robinson can produce if he works 10 hours a day. It is depicted in Figure 32.8A.

The slope of this production possibilities set measures the **marginal rate of transformation**—how much of one good Robinson can get if he decides to sacrifice some of the other good. If Robinson gives up enough labor to produce 1 less pound of fish, he will be able to get 2 more pounds of coconuts. Think about it: if Robinson works one hour less on fish production, he will get 10 pounds less fish. But then if he devotes that time to coconuts, he will get 20 pounds more coconuts. The tradeoff is at a ratio of 2 to 1.

32.10 Comparative Advantage

The construction of the production possibilities set given above was quite simple since there was only one way to produce fish and one way to produce coconuts. What if there is more than one way to produce each good? Suppose that we add another worker to our island economy, who has different skills in producing fish and coconuts.

To be specific, let us call the new worker Friday, and suppose that he can produce 20 pounds of fish per hour, or 10 pounds of coconuts per hour.

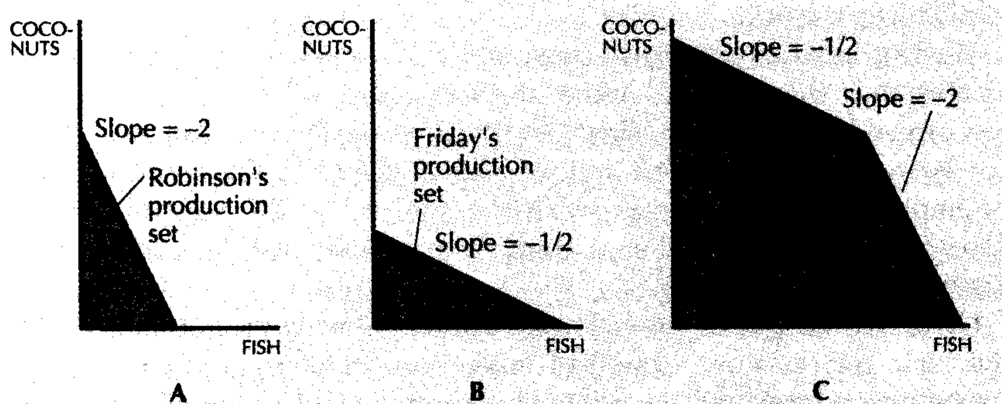
Thus if Friday works for 10 hours, his production possibilities set will be determined by

$$F = 20L_f$$
$$C = 10L_c$$
$$L_c + L_f = 10.$$

Doing the same sort of calculations as we did for Robinson, Friday's production possibilities set is given by

$$\frac{F}{20} + \frac{C}{10} = 10.$$

This is depicted in Figure 32.8B. Note that the marginal rate of transformation between coconuts and fish for Friday is $\Delta C/\Delta F = -1/2$, whereas for Robinson the marginal rate of transformation is -2. For every pound of coconuts that Friday gives up, he can get two pounds of fish; for every pound of fish that Robinson gives up, he can get two pounds of coconuts. In this circumstance we say that Friday has a **comparative advantage** in fish production, and Robinson has a comparative advantage in coconut production. In Figure 32.8 we have depicted three production possibilities sets: Panel A shows Robinson's, panel B shows Friday's, and panel C depicts the joint production possibilities set—how much of each good could be produced in total by both people.



Joint production possibilities. Robinson's and Friday's production possibilities sets and the joint production possibilities set.

The joint production possibilities set combines the best of both workers. If both workers are used entirely to produce coconuts, we will get 300 coconuts—100 from Friday and 200 from Robinson. If we want to get more fish, it makes sense to shift the person who is most productive at fish—Friday—out of coconut production and into fish production. For each pound of coconuts that Friday doesn't produce we get 2 pounds of fish; thus the slope of the joint production possibilities set is $-1/2$—which is exactly Friday's marginal rate of transformation.

When Friday is producing 200 pounds of fish, he is fully occupied. If we want even more fish, we have to switch to using Robinson. From this point on the joint production possibilities set will have a slope of -2, since we will be operating along Robinson's production possibilities set. Finally, if we want to produce as much fish as possible, both Robinson and Friday concentrate on fish production and we get 300 pounds of fish, 200 from Friday, and 100 from Robinson.

Since the workers each have a comparative advantage in different goods, the joint production possibilities set will have a "kink," as shown in Figure 32.8. There is only one kink in this example since there are just two different ways to produce output—Crusoe's way and Friday's way. If there are many different ways to produce output, the production possibilities set will have the more typical "rounded" structure, as depicted in Figure 32.7.

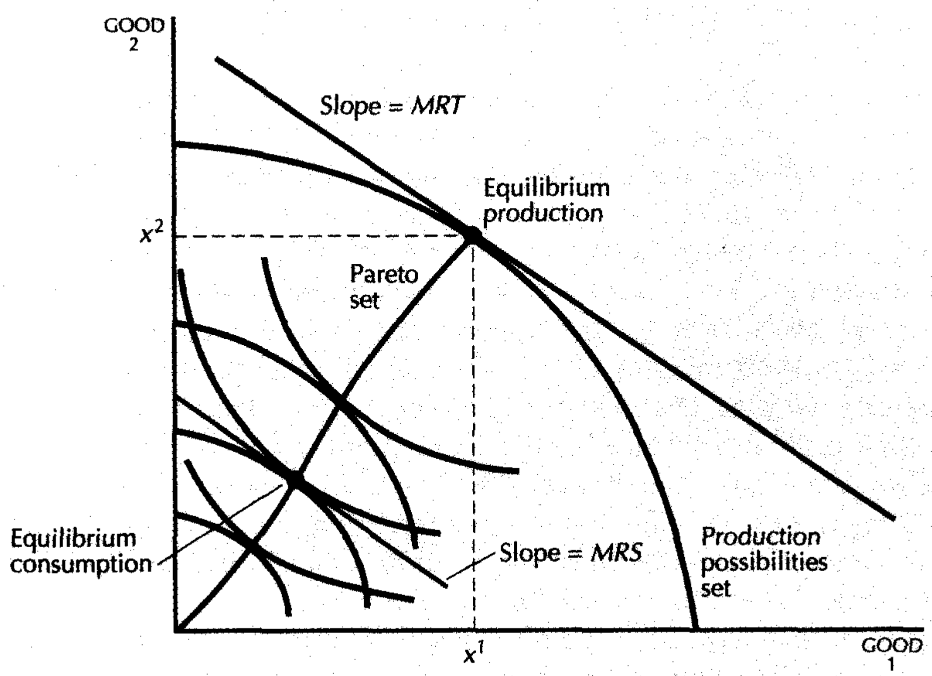
32.11 Pareto Efficiency

In the last two sections we saw how to construct the production possibilities set, the set that describes the feasible consumption bundles for the economy as a whole. Here we consider Pareto efficient ways to choose among the feasible consumption bundles.

We will indicate aggregate consumption bundles by (X^1, X^2). This indicates that there are X^1 units of good 1 and X^2 units of good 2 that are available for consumption. In the Crusoe/Friday economy, the two goods are coconuts and fish, but we will use the (X^1, X^2) notation in order to emphasize the similarities with the analysis in Chapter 31. Once we know the total amount of each good, we can draw an Edgeworth box as in Figure 32.9.

Given (X^1, X^2), the set of Pareto efficient consumption bundles will be the same sort as those examined in the last chapter: the Pareto efficient consumption levels will lie along the Pareto set—the line of mutual tangencies of the indifference curves, as illustrated in Figure 32.9. These are the allocations in which each consumer's marginal rate of substitution—the rate at which he or she is just willing to trade—equals that of the other.

These allocations are Pareto efficient as far as the consumption decisions are concerned. If people can simply trade one good for another, the Pareto set describes the set of bundles that exhausts the gains from trade. But in an economy with production, there is another way to "exchange" one good for another—namely, to produce less of one good and more of another.



Production and the Edgeworth box. At each point on the production possibilities frontier, we can draw an Edgeworth box to illustrate the possible consumption allocations.

The Pareto set describes the set of Pareto efficient bundles *given* the amounts of goods 1 and 2 available, but in an economy with production those amounts can themselves be chosen out of the production possibilities set. Which choices from the production possibilities set will be Pareto efficient choices?

Let us think about the logic underlying the marginal rate of substitution condition. We argued that in a Pareto efficient allocation, the MRS of consumer A had to be equal to the MRS of consumer B: the rate at which consumer A would just be willing to trade one good for the other should be equal to the rate at which consumer B would just be willing to trade one good for the other. If this were not true, then there would be some trade that would make both consumers better off.

Recall that the marginal rate of transformation (MRT) measures the rate at which one good can be "transformed" into the other. Of course, one good really isn't being literally *transformed* into the other. Rather the factors of production are being moved around so as to produce less of one good and more of the other.

Suppose that the economy were operating at a position where the marginal rate of substitution of one of the consumers was not equal to the marginal rate of transformation between the two goods. Then such a position cannot be Pareto efficient. Why? Because at this point, the rate at which the consumer is willing to trade good 1 for good 2 is different from the rate at which good 1 can be transformed into good 2—there is a way to make the consumer better off by rearranging the pattern of production.

Suppose, for example, that the consumer's MRS is 1; the consumer is just willing to substitute good 1 for good 2 on a one-to-one basis. Suppose that the MRT is 2, which means that giving up one unit of good 1 will allow society to produce two units of good 2. Then clearly it makes sense to reduce the production of good 1 by one unit; this will generate two extra units of good 2. Since the consumer was just indifferent between giving up one unit of good 1 and getting one unit of the other good in exchange, he or she will now certainly be better off by getting *two* extra units of good 2.

The same argument can be made whenever one of the consumers has a MRS that is different from the MRT—there will always be a rearrangement of consumption and production that will make that consumer better off. We have already seen that for Pareto efficiency each consumer's MRS should be the same, and the argument given above implies that each consumer's MRS should in fact be equal to the MRT.

Figure 32.9 illustrates a Pareto efficient allocation. The MRSs of each consumer are the same, since their indifference curves are tangent in the Edgeworth box. And each consumer's MRS is equal to the MRT—the slope of the production possibilities set.

32.12 Castaways, Inc.

In the last section we derived the necessary conditions for Pareto efficiency: the MRS of each consumer must equal the MRT. Any way of distributing resources that results in Pareto efficiency must satisfy this condition. Earlier in this chapter, we claimed that a competitive economy with profit-maximizing firms and utility-maximizing consumers would result in a Pareto efficient allocation. In this section we explore the details of how this works.

Our economy now contains two individuals, Robinson and Friday. There are four goods: two factors of production (Robinson's labor and Friday's labor) and two output goods (coconuts and fish). Let us suppose that Robinson and Friday are both shareholders of the firm, which we will now refer to as Castaways, Inc. Of course, they are also the sole employees and customers, but as usual we shall examine each role in turn, and not allow the participants to see the wider picture. After all, the object of the analysis is to understand how a *decentralized* resource allocation system works—one in which each person only has to determine his or her own decisions, without regard for the functioning of the economy as a whole.

Start first with Castaways, Inc., and consider the profit-maximization problem. Castaways, Inc., produces two outputs, coconuts (C) and fish (F), and it uses two kinds of labor, Crusoe's labor (L_C) and Friday's labor

(L_F). Given the price of coconuts (p_C), the price of fish (p_F), and the wage rates of Crusoe and Friday (w_C and w_F), the profit-maximization problem is

$$\max_{C,F,L_F,L_C} p_C C + p_F F - w_C L_C - w_F L_F$$

subject to the technological constraints described by the production possibilities set.

Let us suppose that the firm finds it optimal in equilibrium to hire L_F^* units of Friday's labor and L_C^* units of Crusoe's labor. The question we want to focus on here is how profit maximization determines the pattern of output to produce. Let $L^* = w_C L_C^* + w_F L_F^*$ represent the labor costs of production, and write the profits of the firm, π, as

$$\pi = p_C C + p_F F - L^*.$$

Rearranging this equation, we have

$$C = \frac{\pi + L^*}{p_C} - \frac{p_F F}{p_C}.$$

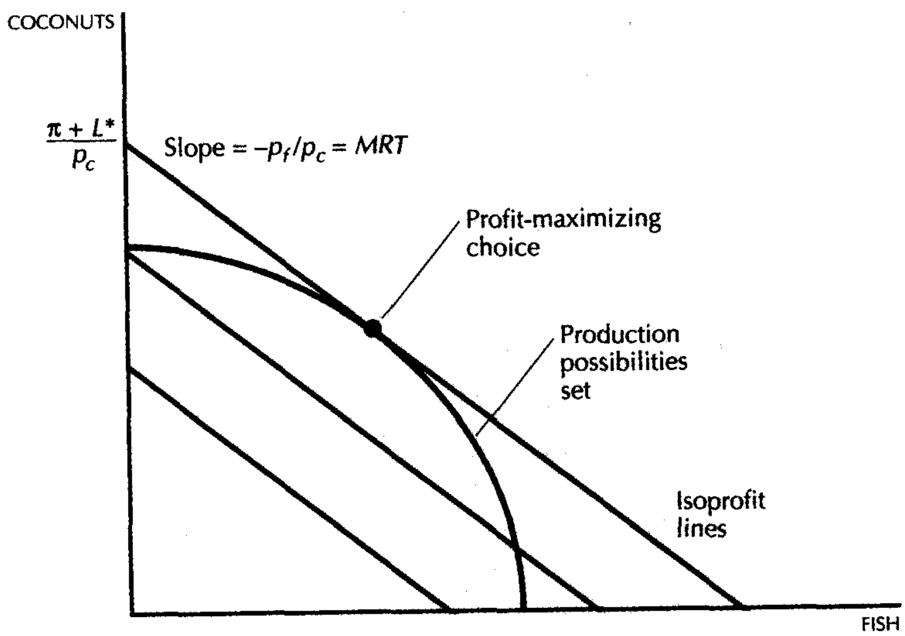
This equation describes the **isoprofit lines** of the firm, as depicted in Figure 32.10, with a slope of $-p_F/p_C$ and a vertical intercept of $(\pi+L^*)/p_C$. Since L^* is fixed by assumption, higher profits will be associated with isoprofit lines that have higher vertical intercepts.

If the firm wants to maximize its profits, it will choose a point on the production possibilities set such that the isoprofit line through that point has the highest possible vertical intercept. By this stage, it should be clear that this implies that the isoprofit line must be tangent to the production possibilities set; that is, that the slope of the production possibilities set (the MRT) should be equal to the slope of the isoprofit line, $-p_F/p_C$:

$$\text{MRT} = -\frac{p_F}{p_C}.$$

We've described this profit-maximization problem in the case of one firm, but it holds for an arbitrary number of firms: each firm that chooses the most profitable way to produce coconuts and fish will operate where the marginal rate of transformation between any two goods equals the price ratio between those two goods. This holds true even if the firms have quite different production possibilities sets, as long as they face the same prices for the two goods.

This means that in equilibrium the prices of the two goods will measure the marginal rate of transformation—the opportunity cost of one good in terms of the other. If you want more coconuts, you will have to give up some fish. How much fish? Just look at the price ratio of fish to coconuts: the ratio of these economic variables tells us what the technological tradeoff must be.



Profit maximization. At the point yielding maximum profits, the marginal rate of transformation must equal the slope of the isoprofit line, $-p_F/p_C$.

32.13 Robinson and Friday as Consumers

We've seen how Castaways, Inc., determines its profit-maximizing production plan. In order to do this, it must hire some labor and it may generate some profits. When it hires labor, it pays wages to the labor; when it makes profits, it pays dividends to its shareholders. Either way the money made by Castaways, Inc., gets paid back to Robinson and Friday, either in the form of wages or profits.

Since the firm pays out all of its receipts to its workers and its shareholders, this means that they must necessarily have enough income to purchase its output. This is just a variation on Walras' law discussed in Chapter 31: people get their income from selling their endowments, so they must always have enough income to purchase those endowments. Here people get income from selling their endowments and also receive profits from the firm. But since money never disappears from or is added to the system, people will always have exactly enough money to purchase what is produced.

What do the consumers do with the money from the firm? As usual, they use the money to purchase consumption goods. Each person chooses the best bundle of goods that he can afford at the prices p_F and p_C. As we've seen earlier, the optimal consumption bundle for each consumer must satisfy the condition that the marginal rate of substitution between the two goods must be equal to the common price ratio. But this price ratio is also equal to the marginal rate of transformation, due to profit maximization by the firm. Thus the necessary conditions for Pareto efficiency are met: the MRS of each consumer equals the MRT.

In this economy, the prices of the goods serve as a signal of relative scarcity. They indicate the technological scarcity—how much the production of one good must be reduced in order to produce more of the other; and they indicate the consumption scarcity—how much people are willing to reduce their consumption of one good in order to acquire some of the other good.

32.14 Decentralized Resource Allocation

The Crusoe-Friday economy is a drastically simplified picture. In order to make a start on a larger model of the functioning of an economy, one needs to use substantially more elaborate mathematics. However, even this simple model contains some useful insights.

The most important of these is the relationship between individuals' *private* goals of utility maximization and the *social* goals of efficient use of resources. Under certain conditions, the individuals' pursuit of private goals will result in an allocation that is Pareto efficient overall. Furthermore, any Pareto efficient allocation can be supported as an outcome of a competitive market, if initial endowments—including the ownership of firms—can be suitably redistributed.

The great virtue of a competitive market is that each individual and each firm only has to worry about its own maximization problem. The only facts that need to be communicated among the firms and the consumers are the prices of the goods. Given these signals of relative scarcity, consumers and firms have enough information to make decisions that achieve an efficient allocation of resources. In this sense, the social problems involved in efficiently utilizing resources can be decentralized, and solved at the individual level.

Each individual can solve his or her own problem of what to consume. The firms face the prices of the goods the consumers consume and decide how much to produce of each of them. In making this decision, they are guided by profit signals. In this context, profits serve as exactly the right guide. To say that a production plan is profitable is to say that people are willing to pay more for some good than it costs to produce it—so it is natural to expand the production of such goods. If all firms pursue a competitive profit-maximizing policy, and all consumers choose consumption bundles to maximize their own utility, then resulting competitive equilibrium must be a Pareto efficient allocation.

Summary

1. The general equilibrium framework can be extended by allowing competitive, profit-maximizing firms to produce goods destined for exchange in the economy.

2. Under certain conditions there exists a set of prices for all of the input and output goods in the economy such that the profit-maximizing actions of firms along with the utility-maximizing behavior of individuals results in the demand for each good equaling the supply in all markets—that is, a competitive equilibrium exists.

3. Under certain conditions the resulting competitive equilibrium will be Pareto efficient: the First Welfare Theorem holds in an economy with production.

4. With the addition of convex production sets, the Second Welfare Theorem also holds in the case of production.

5. When goods are being produced as efficiently as possible, the marginal rate of transformation between two goods indicates the number of units of one good the economy must give up to obtain additional units of the other good.

6. Pareto efficiency requires that each individual's marginal rate of substitution be equal to the marginal rate of transformation.

7. The virtue of competitive markets is that they provide a way to achieve an efficient allocation of resources by decentralizing production and consumption decisions.

REVIEW QUESTIONS

1. The competitive price of coconuts is $6 per pound and the price of fish is $3 per pound. If society were to give up 1 pound of coconuts, how many more pounds of fish could be produced?

2. What would happen if the firm depicted in Figure 32.2 decided to pay a higher wage?

3. In what sense is a competitive equilibrium a good or bad thing for a given economy?

4. If Robinson's marginal rate of substitution between coconuts and fish is -2 and the marginal rate of transformation between the two goods is -1, what should he do if he wants to increase his utility?

5. Suppose that Robinson and Friday both want 60 pounds of fish and 60 pounds of coconuts per day. Using the production rates given in the chapter, how many hours must Robinson and Friday work per day if they don't help each other? Suppose they decide to work together in the most efficient manner possible. Now how many hours each day do they have to work? What is the economic explanation for the reduction in hours?

APPENDIX

Let us derive the calculus conditions for Pareto efficiency in an economy with production. We let X^1 and X^2 represent the total amount of good 1 and good 2 produced and consumed, as in the body of this chapter:

$$X^1 = x_A^1 + x_B^1$$
$$X^2 = x_A^2 + x_B^2.$$

The first thing we need is a convenient way to describe the production possibilities frontier—all the combinations of X^1 and X^2 that are technologically feasible. The most useful way to do this for our purposes is by use of the **transformation function**. This is a function of the aggregate amounts of the two goods $T(X^1, X^2)$, such that the combination (X^1, X^2) is on the production possibilities frontier (the boundary of the production possibilities set) if and only if

$$T(X^1, X^2) = 0.$$

Once we have described the technology, we can calculate the marginal rate of transformation: the rate at which we have to sacrifice good 2 in order to produce more of good 1. Although the name evokes an image of one good being "transformed" into another, that is a somewhat misleading picture. What really happens is that other resources are moved from producing good 2 to producing good 1. Thus, by devoting fewer resources to good 2 and more to good 1, we move from one point on the production possibilities frontier to another. The marginal rate of transformation is just the slope of the production possibilities set, which we denote by dX^2/dX^1.

Consider a small change in production (dX^1, dX^2) that remains feasible. Thus we have

$$\frac{\partial T(X^1, X^2)}{\partial X^1} dX^1 + \frac{\partial T(X^1, X^2)}{\partial X^2} dX^2 = 0.$$

Solving for the marginal rate of transformation:

$$\frac{dX^2}{dX^1} = -\frac{\partial T/\partial X^1}{\partial T/\partial X^2}.$$

We'll use this formula in a moment.

A Pareto efficient allocation is one that maximizes any one person's utility, given the level of the other people's utility. In the two-person case, we can write this maximization problem as

$$\begin{aligned} \max_{x_A^1, x_A^2, x_B^1, x_B^2} \quad & u_A(x_A^1, x_A^2) \\ \text{such that } & u_B(x_B^1, x_B^2) = \overline{u} \\ & T(X^1, X^2) = 0. \end{aligned}$$

The Lagrangian for this problem is

$$\begin{aligned} L = u_A(x_A^1, x_A^2) &- \lambda(u_B(x_B^1, x_B^2) - \overline{u}) \\ &- \mu(T(X_1, X_2) - 0), \end{aligned}$$

and the first-order conditions are

$$\begin{aligned} \frac{\partial L}{\partial x_A^1} &= \frac{\partial u_A}{\partial x_A^1} - \mu \frac{\partial T}{\partial X^1} = 0 \\ \frac{\partial L}{\partial x_A^2} &= \frac{\partial u_A}{\partial x_A^2} - \mu \frac{\partial T}{\partial X^2} = 0 \\ \frac{\partial L}{\partial x_B^1} &= -\lambda \frac{\partial u_B}{\partial x_B^1} - \mu \frac{\partial T}{\partial X^1} = 0 \\ \frac{\partial L}{\partial x_B^2} &= -\lambda \frac{\partial u_B}{\partial x_B^2} - \mu \frac{\partial T}{\partial X^2} = 0. \end{aligned}$$

Rearranging and dividing the first equation by the second gives

$$\frac{\partial u_A / \partial x_A^1}{\partial u_A / \partial x_A^2} = \frac{\partial T / \partial X^1}{\partial T / \partial X^2}.$$

Performing the same operation on the third and fourth equations gives

$$\frac{\partial u_B / \partial x_B^1}{\partial u_B / \partial x_B^2} = \frac{\partial T / \partial X^1}{\partial T / \partial X^2}.$$

The left-hand sides of these equations are our old friends, the marginal rates of substitution. The right-hand side is the marginal rate of transformation. Thus the equations require that each person's marginal rate of substitution between the goods must equal the marginal rate of transformation: the rate at which each person is just willing to substitute one good for the other must be the same as the rate at which it is technologically feasible to transform one good into the other.

The intuition behind this result is straightforward. Suppose that the MRS for some individual was not equal to the MRT. Then the rate at which the individual would be willing to sacrifice one good to get more of the other would be different than the rate that was technologically feasible—but this means that there would be some way to increase that individual's utility while not affecting anyone else's consumption.