CHAPTER 8

SLUTSKY EQUATION

Economists often are concerned with how a consumer's behavior changes in response to changes in the economic environment. The case we want to consider in this chapter is how a consumer's choice of a good responds to changes in its price. It is natural to think that when the price of a good rises the demand for it will fall. However, as we saw in Chapter 6 it is possible to construct examples where the optimal demand for a good *decreases* when its price falls. A good that has this property is called a **Giffen good**.

Giffen goods are pretty peculiar and are primarily a theoretical curiosity, but there are other situations where changes in prices might have "perverse" effects that, on reflection, turn out not to be so unreasonable. For example, we normally think that if people get a higher wage they will work more. But what if your wage went from \$10 an hour to \$1000 an hour? Would you really work more? Might you not decide to work fewer hours and use some of the money you've earned to do other things? What if your wage were \$1,000,000 an hour? Wouldn't you work less?

For another example, think of what happens to your demand for apples when the price goes up. You would probably consume fewer apples. But how about a family who grew apples to sell? If the price of apples went up, their income might go up so much that they would feel that they could now afford to consume more of their own apples. For the consumers in this family, an increase in the price of apples might well lead to an increase in the consumption of apples.

What is going on here? How is it that changes in price can have these ambiguous effects on demand? In this chapter and the next we'll try to sort out these effects.

8.1 The Substitution Effect

When the price of a good changes, there are two sorts of effects: the rate at which you can exchange one good for another changes, and the total purchasing power of your income is altered. If, for example, good 1 becomes cheaper, it means that you have to give up less of good 2 to purchase good 1. The change in the price of good 1 has changed the rate at which the market allows you to "substitute" good 2 for good 1. The trade-off between the two goods that the market presents the consumer has changed.

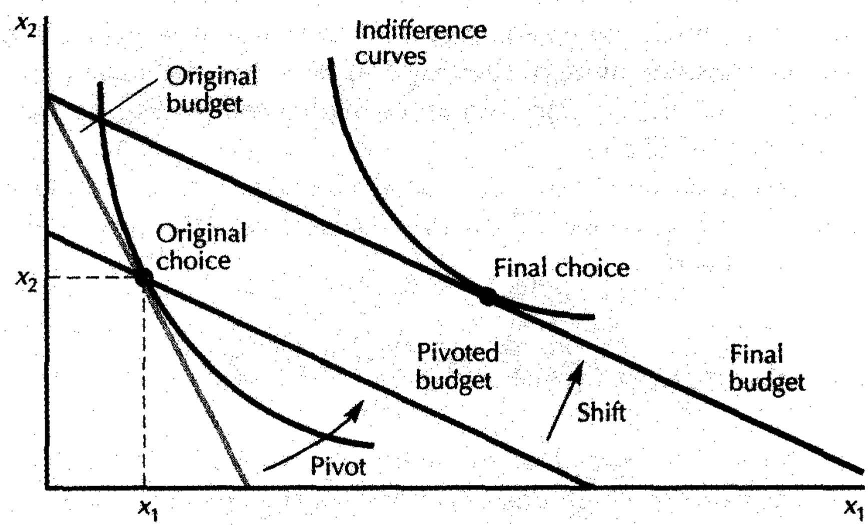
At the same time, if good 1 becomes cheaper it means that your money income will buy more of good 1. The purchasing power of your money has gone up; although the number of dollars you have is the same, the amount that they will buy has increased.

The first part—the change in demand due to the change in the rate of exchange between the two goods—is called the **substitution effect**. The second effect—the change in demand due to having more purchasing power—is called the **income effect**. These are only rough definitions of the two effects. In order to give a more precise definition we have to consider the two effects in greater detail.

The way that we will do this is to break the price movement into two steps: first we will let the *relative* prices change and adjust money income so as to hold purchasing power constant, then we will let purchasing power adjust while holding the relative prices constant.

This is best explained by referring to Figure 8.1. Here we have a situation where the price of good 1 has declined. This means that the budget line rotates around the vertical intercept m/p_2 and becomes flatter. We can break this movement of the budget line up into two steps: first *pivot* the budget line around the *original* demanded bundle and then *shift* the pivoted line out to the *new* demanded bundle.

This "pivot-shift" operation gives us a convenient way to decompose the change in demand into two pieces. The first step—the pivot—is a movement where the slope of the budget line changes while its purchasing power stays constant, while the second step is a movement where the slope stays constant and the purchasing power changes. This decomposition is only a hypothetical construction—the consumer simply observes a change



Pivot and shift. When the price of good 1 changes and income stays fixed, the budget line pivots around the vertical axis. We will view this adjustment as occurring in two stages: first pivot the budget line around the *original* choice, and then shift this line outward to the new demanded bundle.

in price and chooses a new bundle of goods in response. But in analyzing how the consumer's choice changes, it is useful to think of the budget line changing in two stages—first the pivot, then the shift.

What are the economic meanings of the pivoted and the shifted budget lines? Let us first consider the pivoted line. Here we have a budget line with the same slope and thus the same relative prices as the final budget line. However, the money income associated with this budget line is different, since the vertical intercept is different. Since the original consumption bundle (x_1, x_2) lies on the pivoted budget line, that consumption bundle is just affordable. The purchasing power of the consumer has remained constant in the sense that the original bundle of goods is just affordable at the new pivoted line.

Let us calculate how much we have to adjust money income in order to keep the old bundle just affordable. Let m' be the amount of money income that will just make the original consumption bundle affordable; this will be the amount of money income associated with the pivoted budget line. Since (x_1, x_2) is affordable at both (p_1, p_2, m) and (p'_1, p_2, m'), we have

$$m' = p'_1 x_1 + p_2 x_2$$
$$m = p_1 x_1 + p_2 x_2.$$

Subtracting the second equation from the first gives

$$m' - m = x_1[p'_1 - p_1].$$

This equation says that the change in money income necessary to make the old bundle affordable at the new prices is just the original amount of consumption of good 1 times the change in prices.

Letting $\Delta p_1 = p_1' - p_1$ represent the change in price 1, and $\Delta m = m' - m$ represent the change in income necessary to make the old bundle just affordable, we have

$$\Delta m = x_1 \Delta p_1. \tag{8.1}$$

Note that the change in income and the change in price will always move in the same direction: if the price goes up, then we have to raise income to keep the same bundle affordable.

Let's use some actual numbers. Suppose that the consumer is originally consuming 20 candy bars a week, and that candy bars cost 50 cents a piece. If the price of candy bars goes up by 10 cents—so that $\Delta p_1 = .60 - .50 = .10$—how much would income have to change to make the old consumption bundle affordable?

We can apply the formula given above. If the consumer had \$2.00 more income, he would just be able to consume the same number of candy bars, namely, 20. In terms of the formula:

$$\Delta m = \Delta p_1 \times x_1 = .10 \times 20 = \$2.00.$$

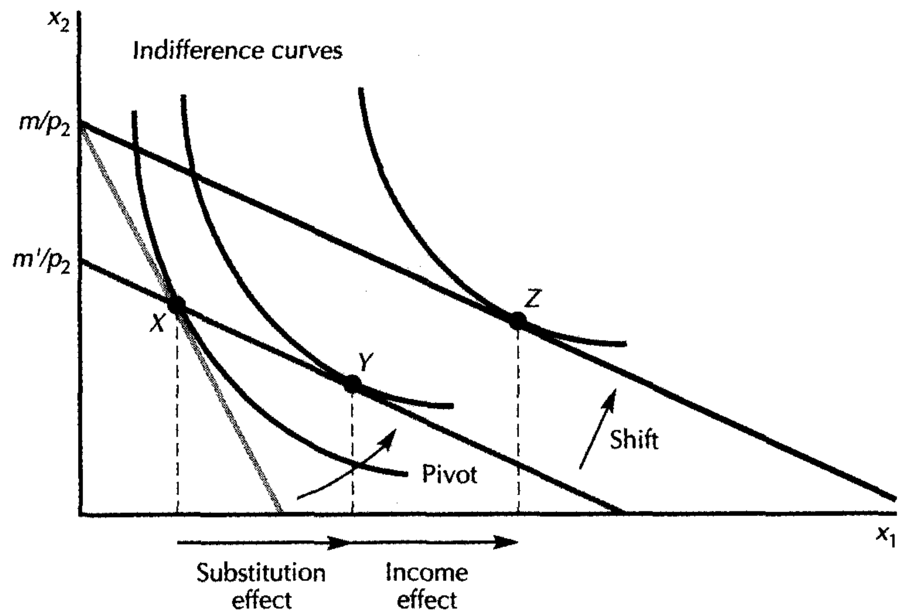
Now we have a formula for the pivoted budget line: it is just the budget line at the new price with income changed by Δm. Note that if the price of good 1 goes down, then the adjustment in income will be negative. When a price goes down, a consumer's purchasing power goes up, so we will have to decrease the consumer's income in order to keep purchasing power fixed. Similarly, when a price goes up, purchasing power goes down, so the change in income necessary to keep purchasing power constant must be positive.

Although (x_1, x_2) is still affordable, it is not generally the optimal purchase at the pivoted budget line. In Figure 8.2 we have denoted the optimal purchase on the pivoted budget line by Y. This bundle of goods is the optimal bundle of goods when we change the price and then adjust dollar income so as to keep the old bundle of goods just affordable. The movement from X to Y is known as the **substitution effect**. It indicates how the consumer "substitutes" one good for the other when a price changes but purchasing power remains constant.

More precisely, the substitution effect, Δx_1^s, is the change in the demand for good 1 when the price of good 1 changes to p_1' and, at the same time, money income changes to m':

$$\Delta x_1^s = x_1(p_1', m') - x_1(p_1, m).$$

In order to determine the substitution effect, we must use the consumer's demand function to calculate the optimal choices at (p_1', m') and (p_1, m). The change in the demand for good 1 may be large or small, depending



Substitution effect and income effect. The pivot gives the substitution effect, and the shift gives the income effect.

on the shape of the consumer's indifference curves. But given the demand function, it is easy to just plug in the numbers to calculate the substitution effect. (Of course the demand for good 1 may well depend on the price of good 2; but the price of good 2 is being held constant during this exercise, so we've left it out of the demand function so as not to clutter the notation.)

The substitution effect is sometimes called the change in **compensated demand**. The idea is that the consumer is being compensated for a price rise by having enough income given back to him to purchase his old bundle. Of course if the price goes down he is "compensated" by having money taken away from him. We'll generally stick with the "substitution" terminology, for consistency, but the "compensation" terminology is also widely used.

EXAMPLE: Calculating the Substitution Effect

Suppose that the consumer has a demand function for milk of the form

$$x_1 = 10 + \frac{m}{10p_1}.$$

Originally his income is \$120 per week and the price of milk is \$3 per quart. Thus his demand for milk will be $10 + 120/(10 \times 3) = 14$ quarts per week.

Now suppose that the price of milk falls to \$2 per quart. Then his demand at this new price will be $10 + 120/(10 \times 2) = 16$ quarts of milk per week. The *total* change in demand is +2 quarts a week.

In order to calculate the substitution effect, we must first calculate how much income would have to change in order to make the original consumption of milk just affordable when the price of milk is \$2 a quart. We apply the formula (8.1):

$$\Delta m = x_1 \Delta p_1 = 14 \times (2 - 3) = -\$14.$$

Thus the level of income necessary to keep purchasing power constant is $m' = m + \Delta m = 120 - 14 = 106$. What is the consumer's demand for milk at the new price, \$2 per quart, and this level of income? Just plug the numbers into the demand function to find

$$x_1(p_1', m') = x_1(2, 106) = 10 + \frac{106}{10 \times 2} = 15.3.$$

Thus the substitution effect is

$$\Delta x_1^s = x_1(2, 106) - x_1(3, 120) = 15.3 - 14 = 1.3.$$

8.2 The Income Effect

We turn now to the second stage of the price adjustment—the shift movement. This is also easy to interpret economically. We know that a parallel shift of the budget line is the movement that occurs when income changes while relative prices remain constant. Thus the second stage of the price adjustment is called the **income effect**. We simply change the consumer's income from m' to m, keeping the prices constant at (p_1', p_2). In Figure 8.2 this change moves us from the point (y_1, y_2) to (z_1, z_2). It is natural to call this last movement the income effect since all we are doing is changing income while keeping the prices fixed at the new prices.

More precisely, the income effect, Δx_1^n, is the change in the demand for good 1 when we change income from m' to m, holding the price of good 1 fixed at p_1':

$$\Delta x_1^n = x_1(p_1', m) - x_1(p_1', m').$$

We have already considered the income effect earlier in section 6.1. There we saw that the income effect can operate either way: it will tend to increase or decrease the demand for good 1 depending on whether we have a normal good or an inferior good.

When the price of a good decreases, we need to decrease income in order to keep purchasing power constant. If the good is a normal good, then this decrease in income will lead to a decrease in demand. If the good is an inferior good, then the decrease in income will lead to an increase in demand.

EXAMPLE: Calculating the Income Effect

In the example given earlier in this chapter we saw that

$$x_1(p_1', m) = x_1(2, 120) = 16$$
$$x_1(p_1', m') = x_1(2, 106) = 15.3.$$

Thus the income effect for this problem is

$$\Delta x_1^n = x_1(2, 120) - x_1(2, 106) = 16 - 15.3 = 0.7.$$

Since milk is a normal good for this consumer, the demand for milk increases when income increases.

8.3 Sign of the Substitution Effect

We have seen above that the income effect can be positive or negative, depending on whether the good is a normal good or an inferior good. What about the substitution effect? If the price of a good goes down, as in Figure 8.2, then the change in the demand for the good due to the substitution effect *must* be nonnegative. That is, if $p_1 > p_1'$, then we *must* have $x_1(p_1', m') \geq x_1(p_1, m)$, so that $\Delta x_1^s \geq 0$.

The proof of this goes as follows. Consider the points on the pivoted budget line in Figure 8.2 where the amount of good 1 consumed is less than at the bundle X. These bundles were all affordable at the old prices (p_1, p_2) but they weren't purchased. Instead the bundle X was purchased. If the consumer is always choosing the best bundle he can afford, then X must be preferred to all of the bundles on the part of the pivoted line that lies inside the original budget set.

This means that the optimal choice on the pivoted budget line must not be one of the bundles that lies underneath the original budget line. The optimal choice on the pivoted line would have to be either X or some point to the right of X. But this means that the new optimal choice must involve consuming at least as much of good 1 as originally, just as we wanted to show. In the case illustrated in Figure 8.2, the optimal choice at the pivoted budget line is the bundle Y, which certainly involves consuming more of good 1 than at the original consumption point, X.

The substitution effect always moves opposite to the price movement. We say that *the substitution effect is negative*, since the change in demand due to the substitution effect is opposite to the change in price: if the price increases, the demand for the good due to the substitution effect decreases.

8.4 The Total Change in Demand

The total change in demand, Δx_1, is the change in demand due to the change in price, holding income constant:

$$\Delta x_1 = x_1(p_1', m) - x_1(p_1, m).$$

We have seen above how this change can be broken up into two changes: the substitution effect and the income effect. In terms of the symbols defined above,

$$\begin{aligned} \Delta x_1 &= \Delta x_1^s + \Delta x_1^n \\ x_1(p_1', m) - x_1(p_1, m) &= [x_1(p_1', m') - x_1(p_1, m)] \\ &\quad + [x_1(p_1', m) - x_1(p_1', m')]. \end{aligned}$$

In words this equation says that the total change in demand equals the substitution effect plus the income effect. This equation is called the **Slutsky identity.**[1] Note that it is an identity: it is true for all values of p_1, p_1', m, and m'. The first and fourth terms on the right-hand side cancel out, so the right-hand side is *identically* equal to the left-hand side.

The content of the Slutsky identity is not just the algebraic identity—that is a mathematical triviality. The content comes in the interpretation of the two terms on the right-hand side: the substitution effect and the income effect. In particular, we can use what we know about the signs of the income and substitution effects to determine the sign of the total effect.

While the substitution effect must always be negative—opposite the change in the price—the income effect can go either way. Thus the total effect may be positive or negative. However, if we have a normal good, then the substitution effect and the income effect work in the same direction. An increase in price means that demand will go down due to the substitution effect. If the price goes up, it is like a decrease in income, which, for a normal good, means a decrease in demand. Both effects reinforce each other. In terms of our notation, the change in demand due to a price increase for a normal good means that

$$\underset{(-)}{\Delta x_1} = \underset{(-)}{\Delta x_1^s} + \underset{(-)}{\Delta x_1^n}.$$

(The minus signs beneath each term indicate that each term in this expression is negative.)

[1] Named for Eugen Slutsky (1880–1948), a Russian economist who investigated demand theory.

Note carefully the sign on the income effect. Since we are considering a situation where the price rises, this implies a decrease in purchasing power—for a normal good this will imply a decrease in demand.

On the other hand, if we have an inferior good, it might happen that the income effect outweighs the substitution effect, so that the total change in demand associated with a price increase is actually positive. This would be a case where

$$\underset{(?)}{\Delta x_1} = \underset{(-)}{\Delta x_1^s} + \underset{(+)}{\Delta x_1^n}.$$

If the second term on the right-hand side—the income effect—is large enough, the total change in demand could be positive. This would mean that an increase in price could result in an *increase* in demand. This is the perverse Giffen case described earlier: the increase in price has reduced the consumer's purchasing power so much that he has increased his consumption of the inferior good.

But the Slutsky identity shows that this kind of perverse effect can only occur for inferior goods: if a good is a normal good, then the income and substitution effects reinforce each other, so that the total change in demand is always in the "right" direction.

Thus a Giffen good must be an inferior good. But an inferior good is not necessarily a Giffen good: the income effect not only has to be of the "wrong" sign, it also has to be large enough to outweigh the "right" sign of the substitution effect. This is why Giffen goods are so rarely observed in real life: they would not only have to be inferior goods, but they would have to be *very* inferior.

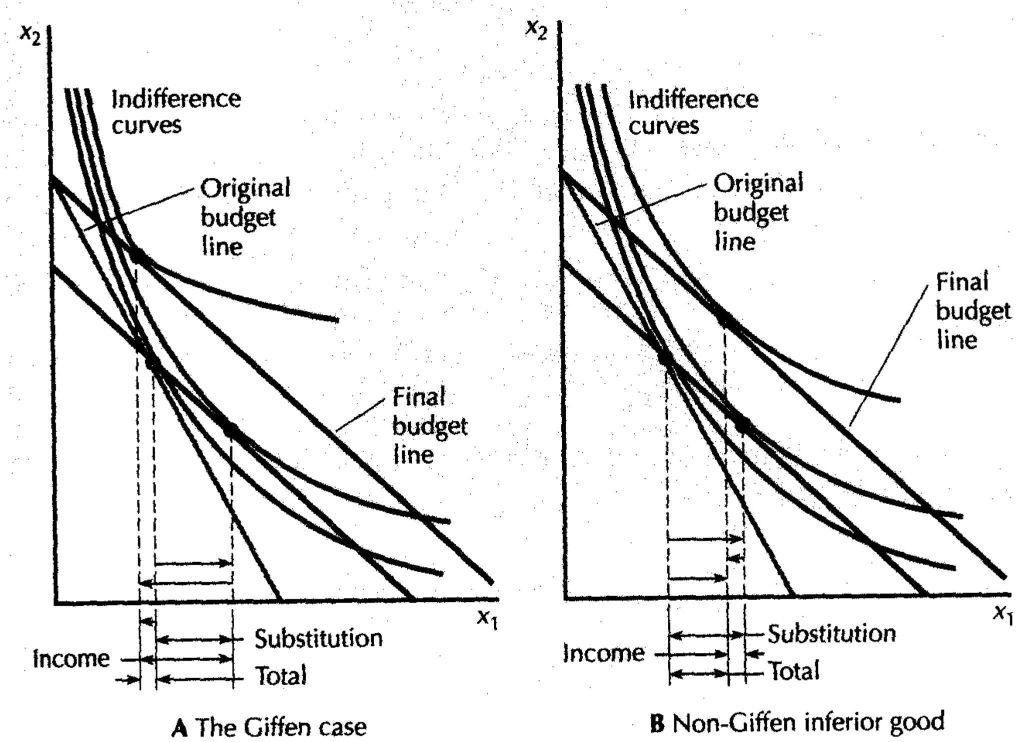
This is illustrated graphically in Figure 8.3. Here we illustrate the usual pivot-shift operation to find the substitution effect and the income effect. In both cases, good 1 is an inferior good, and the income effect is therefore negative. In Figure 8.3A, the income effect is large enough to outweigh the substitution effect and produce a Giffen good. In Figure 8.3B, the income effect is smaller, and thus good 1 responds in the ordinary way to the change in its price.

8.5 Rates of Change

We have seen that the income and substitution effects can be described graphically as a combination of pivots and shifts, or they can be described algebraically in the Slutsky identity

$$\Delta x_1 = \Delta x_1^s + \Delta x_1^n,$$

which simply says that the total change in demand is the substitution effect plus the income effect. The Slutsky identity here is stated in terms



Inferior goods. Panel A shows a good that is inferior enough to cause the Giffen case. Panel B shows a good that is inferior, but the effect is not strong enough to create a Giffen good.

Figure 8.3

of absolute changes, but it is more common to express it in terms of *rates* of change.

When we express the Slutsky identity in terms of rates of change it turns out to be convenient to define Δx_1^m to be the *negative* of the income effect:

$$\Delta x_1^m = x_1(p_1', m') - x_1(p_1', m) = -\Delta x_1^n.$$

Given this definition, the Slutsky identity becomes

$$\Delta x_1 = \Delta x_1^s - \Delta x_1^m.$$

If we divide each side of the identity by Δp_1, we have

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1^m}{\Delta p_1}. \tag{8.2}$$

The first term on the right-hand side is the rate of change of demand when price changes and income is adjusted so as to keep the old bundle affordable—the substitution effect. Let's work on the second term. Since we have an income change in the numerator, it would be nice to get an income change in the denominator.

Remember that the income change, Δm, and the price change, Δp_1, are related by the formula

$$\Delta m = x_1 \Delta p_1.$$

Solving for Δp_1 we find

$$\Delta p_1 = \frac{\Delta m}{x_1}.$$

Now substitute this expression into the last term in (8.2) to get our final formula:

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1^m}{\Delta m} x_1.$$

This is the Slutsky identity in terms of rates of change. We can interpret each term as follows:

$$\frac{\Delta x_1}{\Delta p_1} = \frac{x_1(p_1', m) - x_1(p_1, m)}{\Delta p_1}$$

is the rate of change in demand as price changes, holding income fixed;

$$\frac{\Delta x_1^s}{\Delta p_1} = \frac{x_1(p_1', m') - x_1(p_1, m)}{\Delta p_1}$$

is the rate of change in demand as the price changes, adjusting income so as to keep the old bundle just affordable, that is, the substitution effect; and

$$\frac{\Delta x_1^m}{\Delta m} x_1 = \frac{x_1(p_1', m') - x_1(p_1', m)}{m' - m} x_1 \tag{8.3}$$

is the rate of change of demand holding prices fixed and adjusting income, that is, the income effect.

The income effect is itself composed of two pieces: how demand changes as income changes, times the original level of demand. When the price changes by Δp_1, the change in demand due to the income effect is

$$\Delta x_1^m = \frac{x_1(p_1', m') - x_1(p_1', m)}{\Delta m} x_1 \Delta p_1.$$

But this last term, $x_1 \Delta p_1$, is just the change in income necessary to keep the old bundle feasible. That is, $x_1 \Delta p_1 = \Delta m$, so the change in demand due to the income effect reduces to

$$\Delta x_1^m = \frac{x_1(p_1', m') - x_1(p_1', m)}{\Delta m} \Delta m,$$

just as we had before.

8.6 The Law of Demand

In Chapter 5 we voiced some concerns over the fact that consumer theory seemed to have no particular content: demand could go up or down when a price increased, and demand could go up or down when income increased. If a theory doesn't restrict observed behavior in *some* fashion it isn't much of a theory. A model that is consistent with all behavior has no real content.

However, we know that consumer theory does have some content—we've seen that choices generated by an optimizing consumer must satisfy the Strong Axiom of Revealed Preference. Furthermore, we've seen that any price change can be decomposed into two changes: a substitution effect that is sure to be negative—opposite the direction of the price change—and an income effect whose sign depends on whether the good is a normal good or an inferior good.

Although consumer theory doesn't restrict how demand changes when price changes or how demand changes when income changes, it does restrict how these two kinds of changes interact. In particular, we have the following.

The Law of Demand. *If the demand for a good increases when income increases, then the demand for that good must decrease when its price increases.*

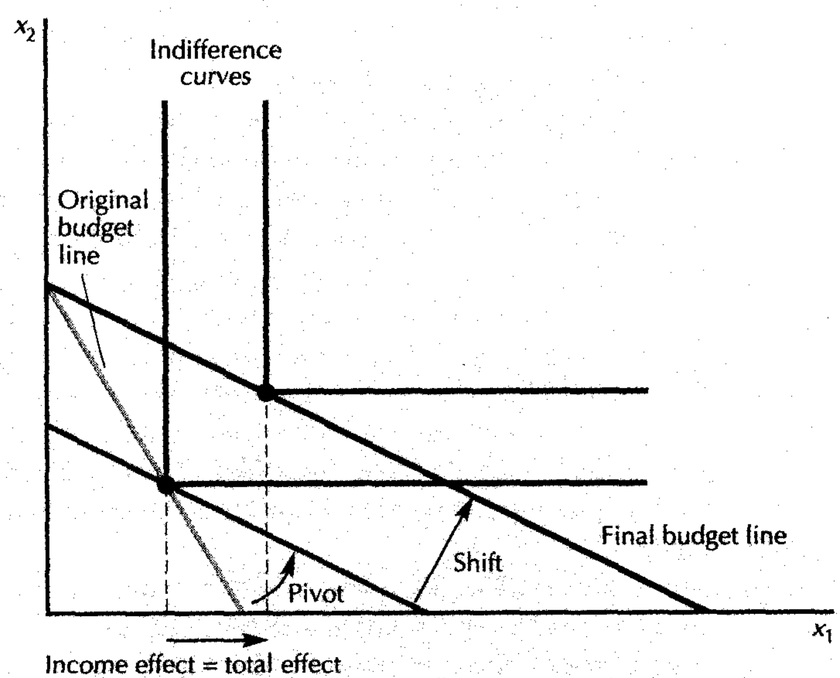
This follows directly from the Slutsky equation: if the demand increases when income increases, we have a normal good. And if we have a normal good, then the substitution effect and the income effect reinforce each other, and an increase in price will unambiguously reduce demand.

8.7 Examples of Income and Substitution Effects

Let's now consider some examples of price changes for particular kinds of preferences and decompose the demand changes into the income and the substitution effects.

We start with the case of perfect complements. The Slutsky decomposition is illustrated in Figure 8.4. When we pivot the budget line around the chosen point, the optimal choice at the new budget line is the same as at the old one—this means that the substitution effect is zero. The change in demand is due entirely to the income effect.

What about the case of perfect substitutes, illustrated in Figure 8.5? Here when we tilt the budget line, the demand bundle jumps from the vertical axis to the horizontal axis. There is no shifting left to do! The entire change in demand is due to the substitution effect.



Perfect complements. Slutsky decomposition with perfect complements.

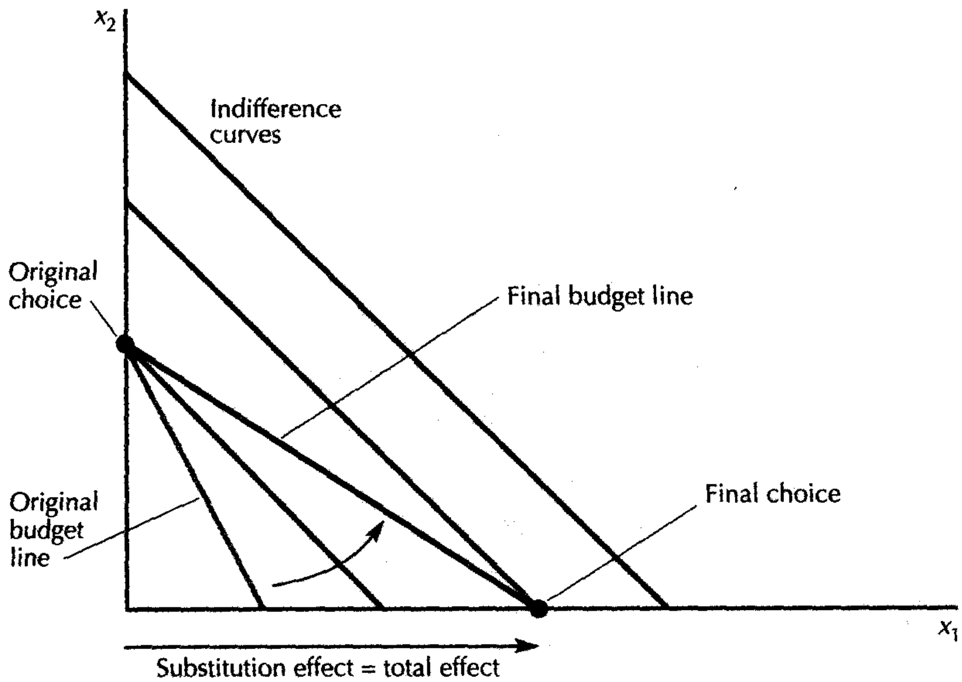
As a third example, let us consider the case of quasilinear preferences. This situation is somewhat peculiar. We have already seen that a shift in income causes no change in demand for good 1 when preferences are quasilinear. This means that the entire change in demand for good 1 is due to the substitution effect, and that the income effect is zero, as illustrated in Figure 8.6.

EXAMPLE: Rebating a Tax

In 1974 the Organization of Petroleum Exporting Countries (OPEC) instituted an oil embargo against the United States. OPEC was able to stop oil shipments to U.S. ports for several weeks. The vulnerability of the United States to such disruptions was very disturbing to Congress and the president, and there were many plans proposed to reduce the United States's dependence on foreign oil.

One such plan involved increasing the gasoline tax. Increasing the cost of gasoline to the consumers would make them reduce their consumption of gasoline, and the reduced demand for gasoline would in turn reduce the demand for foreign oil.

But a straight increase in the tax on gasoline would hit consumers where it hurts—in the pocketbook—and by itself such a plan would be politically



Perfect substitutes. Slutsky decomposition with perfect substitutes.

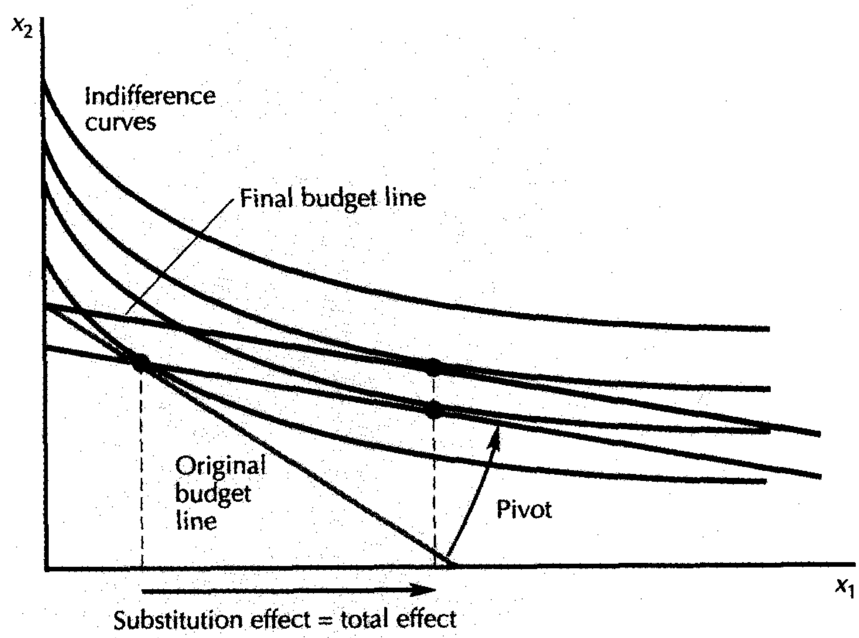
infeasible. So it was suggested that the revenues raised from consumers by this tax would be returned to the consumers in the form of direct money payments, or via the reduction of some other tax.

Critics of this proposal argued that paying the revenue raised by the tax back to the consumers would have no effect on demand since they could just use the rebated money to purchase more gasoline. What does economic analysis say about this plan?

Let us suppose, for simplicity, that the tax on gasoline would end up being passed along entirely to the consumers of gasoline so that the price of gasoline will go up by exactly the amount of the tax. (In general, only part of the tax would be passed along, but we will ignore that complication here.) Suppose that the tax would raise the price of gasoline from p to $p' = p + t$, and that the average consumer would respond by reducing his demand from x to x'. The average consumer is paying t dollars more for gasoline, and he is consuming x' gallons of gasoline after the tax is imposed, so the amount of revenue raised by the tax from the average consumer would be

$$R = tx' = (p' - p)x'.$$

Note that the revenue raised by the tax will depend on how much gasoline the consumer *ends up* consuming, x', not how much he was initially



Quasilinear preferences. In the case of quasilinear preferences, the entire change in demand is due to the substitution effect.

consuming, x.

If we let y be the expenditure on all other goods and set its price to be 1, then the original budget constraint is

$$px + y = m, \tag{8.4}$$

and the budget constraint in the presence of the tax-rebate plan is

$$(p + t)x' + y' = m + tx'. \tag{8.5}$$

In budget constraint (8.5) the average consumer is choosing the left-hand side variables—the consumption of each good—but the right-hand side—his income and the rebate from the government—are taken as fixed. The rebate depends on what all consumers do, not what the average consumer does. In this case, the rebate turns out to be the taxes collected from the average consumer—but that's because he is average, not because of any causal connection.

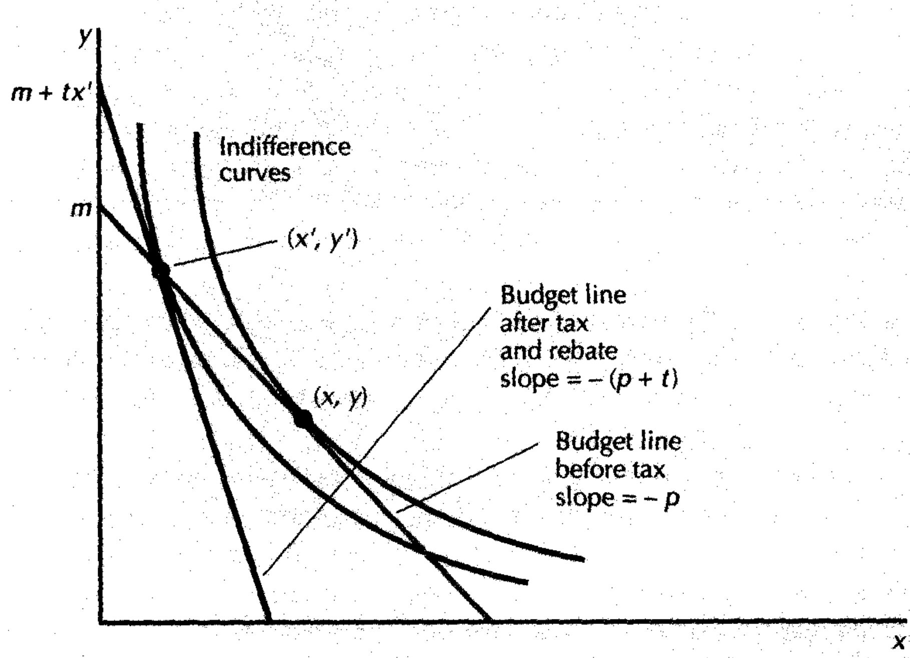
If we cancel tx' from each side of equation (8.5), we have

$$px' + y' = m.$$

Thus (x', y') is a bundle that was affordable under the original budget constraint and rejected in favor of (x, y). Thus it must be that (x, y)

is preferred to (x', y'): the consumers are made worse off by this plan. Perhaps that is why it was never put into effect!

The equilibrium with a rebated tax is depicted in Figure 8.7. The tax makes good 1 more expensive, and the rebate increases money income. The original bundle is no longer affordable, and the consumer is definitely made worse off. The consumer's choice under the tax-rebate plan involves consuming less gasoline and more of "all other goods."



Rebating a tax. Taxing a consumer and rebating the tax revenues makes the consumer worse off.

What can we say about the amount of consumption of gasoline? The average consumer could afford his old consumption of gasoline, but because of the tax, gasoline is now more expensive. In general, the consumer would choose to consume less of it.

EXAMPLE: Voluntary Real Time Pricing

Electricity production suffers from an extreme capacity problem: it is relatively cheap to produce up to capacity, at which point it is, by definition, impossible to produce more. Building capacity is extremely expensive, so finding ways to reduce the use of electricity during periods of peak demand is very attractive from an economic point of view.

In states with warm climates, such as Georgia, roughly 30 percent of usage during periods of peak demand is due to air conditioning. Furthermore, it is relatively easy to forecast temperature one day ahead so that potential users will have time to adjust their demand by setting their air conditioning to a higher temperature, wearing light clothes, and so on. The challenge is to set up a pricing system so that those users who are able to cut back on their electricity use will have an incentive to reduce their consumption.

One way to accomplish this is through the use of Real Time Pricing (RTP). In a Real Time Pricing program, large industrial users are equipped with special meters that allow the price of electricity to vary from minute to minute, depending on signals sent from the electricity generating company. As the demand for electricity approaches capacity, the generating company increases the price so as to encourage users to cut back on their usage. The price schedule is determined as a function of the total demand for electricity.

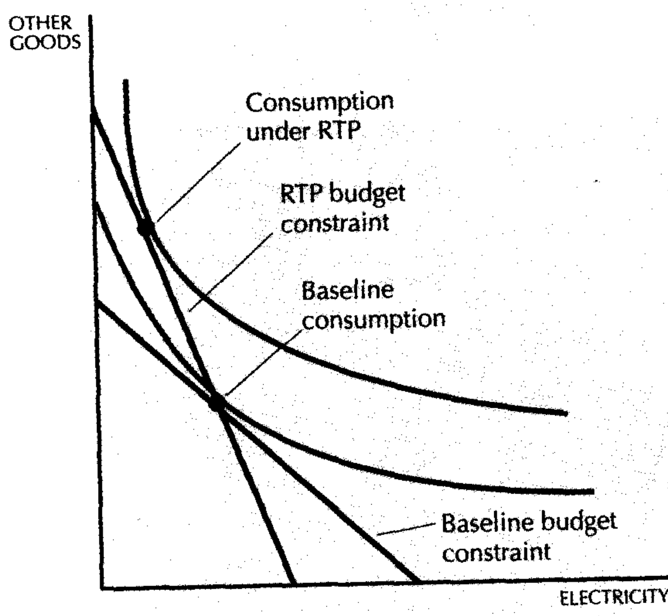
Georgia Power Company claims that it runs the largest real time pricing program in the world. In 1999 it was able to reduce demand by 750 megawatts on high-price days by inducing some large customers to cut their demand by as much as 60 percent.

Georgia Power has devised several interesting variations on the basic real time pricing model. In one pricing plan, customers are assigned a baseline quantity, which represents their normal usage. When electricity is in short supply and the real time price increases, these users face a higher price for electricity use in excess of their baseline quantity. But they also receive a *rebate* if they can manage to cut their electricity use below their baseline amount.

Figure 8.8 shows how this affects the budget line of the users. The vertical axis is "money to spend on things other than electricity" and the horizontal axis is "electricity use." In normal times, users choose their electricity consumption to maximize utility subject to a budget constraint which is determined by the baseline price of electricity. The resulting choice is their baseline consumption.

When the temperature rises, the real time price increases, making electricity more expensive. But this increase in price is a good thing for users who can cut back their consumption, since they receive a rebate based on the high real time price for every kilowatt of reduced usage. If usage stays at the baseline amount, then the user's bill will not change.

It is not hard to see that this pricing plan is a Slutsky pivot around the baseline consumption. Thus we can be confident that electricity usage will decline, and that users will be at least as well off at the real time price as at the baseline price. Indeed, the program has been quite popular, with over 1,600 voluntary participants.



Voluntary real time pricing. Users pay higher rates for additional electricity when the real time price rises, but they also get rebates at the same price if they cut back their use. This results in a pivot around the baseline use and tends to make the customers better off.

8.8 Another Substitution Effect

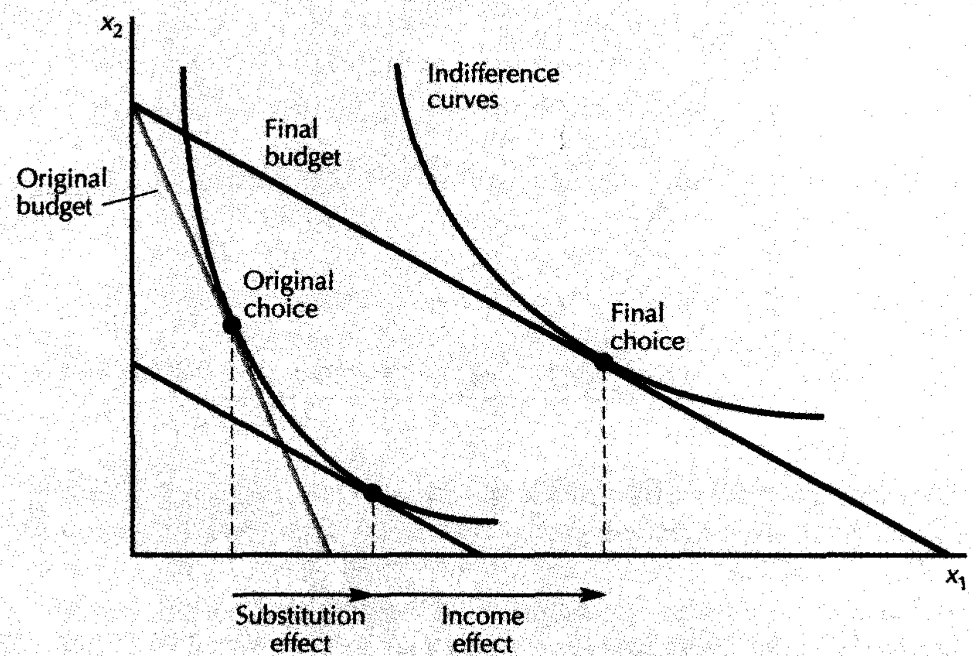
The substitution effect is the name that economists give to the change in demand when prices change but a consumer's purchasing power is held constant, so that the original bundle remains affordable. At least this is *one* definition of the substitution effect. There is another definition that is also useful.

The definition we have studied above is called the **Slutsky substitution effect**. The definition we will describe in this section is called the **Hicks substitution effect.**[2]

Suppose that instead of pivoting the budget line around the original consumption bundle, we now *roll* the budget line around the indifference curve through the original consumption bundle, as depicted in Figure 8.9. In this way we present the consumer with a new budget line that has the same relative prices as the final budget line but has a different income. The purchasing power he has under this budget line will no longer be sufficient to

2 The concept is named for Sir John Hicks, an English recipient of the Nobel Prize in Economics.

purchase his original bundle of goods—but it will be sufficient to purchase a bundle that is just *indifferent* to his original bundle.



The Hicks substitution effect. Here we pivot the budget line around the indifference curve rather than around the original choice.

Thus the Hicks substitution effect keeps *utility* constant rather than keeping purchasing power constant. The Slutsky substitution effect gives the consumer just enough money to get back to his old level of consumption, while the Hicks substitution effect gives the consumer just enough money to get back to his old indifference curve. Despite this difference in definition, it turns out that the Hicks substitution effect must be negative—in the sense that it is in a direction opposite that of the price change—just like the Slutsky substitution effect.

The proof is again by revealed preference. Let (x_1, x_2) be a demanded bundle at some prices (p_1, p_2), and let (y_1, y_2) be a demanded bundle at some other prices (q_1, q_2). Suppose that income is such that the consumer is indifferent between (x_1, x_2) and (y_1, y_2). Since the consumer is indifferent between (x_1, x_2) and (y_1, y_2), neither bundle can be revealed preferred to the other.

Using the definition of revealed preference, this means that the following two inequalities are *not* true:

$$p_1 x_1 + p_2 x_2 > p_1 y_1 + p_2 y_2$$

$$q_1 y_1 + q_2 y_2 > q_1 x_1 + q_2 x_2.$$

It follows that these inequalities *are* true:

$$p_1 x_1 + p_2 x_2 \leq p_1 y_1 + p_2 y_2$$

$$q_1 y_1 + q_2 y_2 \leq q_1 x_1 + q_2 x_2.$$

Adding these inequalities together and rearranging them we have

$$(q_1 - p_1)(y_1 - x_1) + (q_2 - p_2)(y_2 - x_2) \leq 0.$$

This is a general statement about how demands change when prices change if income is adjusted so as to keep the consumer on the same indifference curve. In the particular case we are concerned with, we are only changing the first price. Therefore $q_2 = p_2$, and we are left with

$$(q_1 - p_1)(y_1 - x_1) \leq 0.$$

This equation says that the change in the quantity demanded must have the opposite sign from that of the price change, which is what we wanted to show.

The total change in demand is still equal to the substitution effect plus the income effect—but now it is the Hicks substitution effect. Since the Hicks substitution effect is also negative, the Slutsky equation takes exactly the same form as we had earlier and has exactly the same interpretation. Both the Slutsky and Hicks definitions of the substitution effect have their place, and which is more useful depends on the problem at hand. It can be shown that for small changes in price, the two substitution effects are virtually identical.

8.9 Compensated Demand Curves

We have seen how the quantity demanded changes as a price changes in three different contexts: holding income fixed (the standard case), holding purchasing power fixed (the Slutsky substitution effect), and holding utility fixed (the Hicks substitution effect). We can draw the relationship between price and quantity demanded holding any of these three variables fixed. This gives rise to three different demand curves: the standard demand curve, the Slutsky demand curve, and the Hicks demand curve.

The analysis of this chapter shows that the Slutsky and Hicks demand curves are always downward sloping curves. Furthermore the ordinary demand curve is a downward sloping curve for normal goods. However, the Giffen analysis shows that it is theoretically possible that the ordinary demand curve may slope upwards for an inferior good.

The Hicksian demand curve—the one with utility held constant—is sometimes called the **compensated demand curve.** This terminology arises naturally if you think of constructing the Hicksian demand curve by adjusting income as the price changes so as to keep the consumer's utility constant. Hence the consumer is "compensated" for the price changes, and his utility is the same at every point on the Hicksian demand curve. This is in contrast to the situation with an ordinary demand curve. In this case the consumer is worse off facing higher prices than lower prices since his income is constant.

The compensated demand curve turns out to be very useful in advanced courses, especially in treatments of benefit-cost analysis. In this sort of analysis it is natural to ask what size payments are necessary to compensate consumers for some policy change. The magnitude of such payments gives a useful estimate of the cost of the policy change. However, actual calculation of compensated demand curves requires more mathematical machinery than we have developed in this text.

Summary

1. When the price of a good decreases, there will be two effects on consumption. The change in relative prices makes the consumer want to consume more of the cheaper good. The increase in purchasing power due to the lower price may increase or decrease consumption, depending on whether the good is a normal good or an inferior good.

2. The change in demand due to the change in relative prices is called the substitution effect; the change due to the change in purchasing power is called the income effect.

3. The substitution effect is how demand changes when prices change and purchasing power is held constant, in the sense that the original bundle remains affordable. To hold real purchasing power constant, money income will have to change. The necessary change in money income is given by $\Delta m = x_1 \Delta p_1$.

4. The Slutsky equation says that the total change in demand is the sum of the substitution effect and the income effect.

5. The Law of Demand says that normal goods must have downward-sloping demand curves.

REVIEW QUESTIONS

1. Suppose a consumer has preferences between two goods that are perfect substitutes. Can you change prices in such a way that the entire demand response is due to the income effect?

2. Suppose that preferences are concave. Is it still the case that the substitution effect is negative?

3. In the case of the gasoline tax, what would happen if the rebate to the consumers were based on their original consumption of gasoline, x, rather than on their final consumption of gasoline, x'?

4. In the case described in the preceding question, would the government be paying out more or less than it received in tax revenues?

5. In this case would the consumers be better off or worse off if the tax with rebate based on original consumption were in effect?

APPENDIX

Let us derive the Slutsky equation using calculus. Consider the Slutsky definition of the substitution effect, in which the income is adjusted so as to give the consumer just enough to buy the original consumption bundle, which we will now denote by $(\overline{x}_1, \overline{x}_2)$. If the prices are (p_1, p_2), then the consumer's actual choice with this adjustment will depend on (p_1, p_2) and $(\overline{x}_1, \overline{x}_2)$. Let's call this relationship the **Slutsky demand function** for good 1, and write it as $x_1^s(p_1, p_2, \overline{x}_1, \overline{x}_2)$.

Suppose the original demanded bundle is $(\overline{x}_1, \overline{x}_2)$ at prices $(\overline{p}_1, \overline{p}_2)$ and income $\overline{m}$. The Slutsky demand function tells us what the consumer would demand facing some different prices (p_1, p_2) and having income $p_1\overline{x}_1 + p_2\overline{x}_2$. Thus the Slutsky demand function at $(p_1, p_2, \overline{x}_1, \overline{x}_2)$ is the ordinary demand at (p_1, p_2) and income $p_1\overline{x}_1 + p_2\overline{x}_2$. That is,

$$x_1^s(p_1, p_2, \overline{x}_1, \overline{x}_2) \equiv x_1(p_1, p_2, p_1\overline{x}_1 + p_2\overline{x}_2).$$

This equation says that the Slutsky demand at prices (p_1, p_2) is that amount which the consumer would demand if he had enough income to purchase his original bundle of goods $(\overline{x}_1, \overline{x}_2)$. This is just the definition of the Slutsky demand function.

Differentiating this identity with respect to p_1, we have

$$\frac{\partial x_1^s(p_1, p_2, \overline{x}_1, \overline{x}_2)}{\partial p_1} = \frac{\partial x_1(p_1, p_2, \overline{m})}{\partial p_1} + \frac{\partial x_1(p_1, p_2, \overline{m})}{\partial m}\overline{x}_1.$$

Rearranging we have

$$\frac{\partial x_1(p_1, p_2, \overline{m})}{\partial p_1} = \frac{\partial x_1^s(p_1, p_2, \overline{x}_1, \overline{x}_2)}{\partial p_1} - \frac{\partial x_1(p_1, p_2, \overline{m})}{\partial m}\overline{x}_1.$$

Note the use of the chain rule in this calculation.

This is a derivative form of the Slutsky equation. It says that the total effect of a price change is composed of a substitution effect (where income is adjusted to keep the bundle $(\overline{x}_1, \overline{x}_2)$ feasible) and an income effect. We know from the text that the substitution effect is negative and that the sign of the income effect depends on whether the good in question is inferior or not. As you can see, this is just the form of the Slutsky equation considered in the text, except that we have replaced the Δ's with derivative signs.

What about the Hicks substitution effect? It is also possible to define a Slutsky equation for it. We let $x_1^h(p_1, p_2, \overline{u})$ be the Hicksian demand function, which measures how much the consumer demands of good 1 at prices (p_1, p_2) if income is adjusted to keep the level of *utility* constant at the original level $\overline{u}$. It turns out that in this case the Slutsky equation takes the form

$$\frac{\partial x_1(p_1, p_2, m)}{\partial p_1} = \frac{\partial x_1^h(p_1, p_2, \overline{u})}{\partial p_1} - \frac{\partial x_1(p_1, p_2, m)}{\partial m}\overline{x}_1.$$

The proof of this equation hinges on the fact that

$$\frac{\partial x_1^h(p_1, p_2, \overline{u})}{\partial p_1} = \frac{\partial x_1^s(p_1, p_2, \overline{x}_1, \overline{x}_2)}{\partial p_1}$$

for infinitesimal changes in price. That is, for derivative size changes in price, the Slutsky substitution and the Hicks substitution effect are the same. The proof of this is not terribly difficult, but it involves some concepts that are beyond the scope of this book. A relatively simple proof is given in Hal R. Varian, *Microeconomic Analysis,* 3rd ed. (New York: Norton, 1992).

EXAMPLE: Rebating a Small Tax

We can use the calculus version of the Slutsky equation to see how consumption choices would react to a small change in a tax when the tax revenues are rebated to the consumers.

Assume, as before, that the tax causes the price to rise by the full amount of the tax. Let x be the amount of gasoline, p its original price, and t the amount of the tax. Then the change in consumption will be given by

$$dx = \frac{\partial x}{\partial p}t + \frac{\partial x}{\partial m}tx.$$

The first term measures how demand responds to the price change times the amount of the price change—which gives us the price effect of the tax. The second terms tells us how demand responds to a change in income times the amount that income has changed—income has gone up by the amount of the tax revenues rebated to the consumer.

Now use Slutsky's equation to expand the first term on the right-hand side to get the substitution and income effects of the price change itself:

$$dx = \frac{\partial x^s}{\partial p}t - \frac{\partial x}{\partial m}tx + \frac{\partial x}{\partial m}tx = \frac{\partial x^s}{\partial p}t.$$

The income effect cancels out, and all that is left is the pure substitution effect. Imposing a small tax and rebating the revenues of the tax is just like imposing a price change and adjusting income so that the old consumption bundle is feasible—as long as the tax is small enough so that the derivative approximation is valid.