CHAPTER 9

BUYING AND SELLING

In the simple model of the consumer that we considered in the preceding chapters, the income of the consumer was given. In reality people earn their income by selling things that they own: items that they have produced, assets that they have accumulated, or, most commonly, their own labor. In this chapter we will examine how the earlier model must be modified so as to describe this kind of behavior.

9.1 Net and Gross Demands

As before, we will limit ourselves to the two-good model. We now suppose that the consumer starts off with an **endowment** of the two goods, which we will denote by (ω_1, ω_2).[1] This is how much of the two goods the consumer has *before* he enters the market. Think of a farmer who goes to market with ω_1 units of carrots and ω_2 units of potatoes. The farmer inspects the prices available at the market and decides how much he wants to buy and sell of the two goods.

[1] The Greek letter ω, omega, is pronounced "o–may–gah."

Let us make a distinction here between the consumer's **gross demands** and his **net demands**. The gross demand for a good is the amount of the good that the consumer actually ends up consuming: how much of each of the goods he or she takes home from the market. The net demand for a good is the *difference* between what the consumer ends up with (the gross demand) and the initial endowment of goods. The net demand for a good is simply the amount that is bought or sold of the good.

If we let (x_1, x_2) be the gross demands, then $(x_1 - \omega_1, x_2 - \omega_2)$ are the net demands. Note that while the gross demands are typically positive numbers, the net demands may be positive or negative. If the net demand for good 1 is negative, it means that the consumer wants to consume less of good 1 than she has; that is, she wants to *supply* good 1 to the market. A negative net demand is simply an amount supplied.

For purposes of economic analysis, the gross demands are the more important, since that is what the consumer is ultimately concerned with. But the net demands are what are actually exhibited in the market and thus are closer to what the layman means by demand or supply.

9.2 The Budget Constraint

The first thing we should do is to consider the form of the budget constraint. What constrains the consumer's final consumption? It must be that the value of the bundle of goods that she goes home with must be equal to the value of the bundle of goods that she came with. Or, algebraically:

$$p_1 x_1 + p_2 x_2 = p_1 \omega_1 + p_2 \omega_2.$$

We could just as well express this budget line in terms of net demands as

$$p_1(x_1 - \omega_1) + p_2(x_2 - \omega_2) = 0.$$

If $(x_1 - \omega_1)$ is positive we say that the consumer is a **net buyer** or **net demander** of good 1; if it is negative we say that she is a **net seller** or **net supplier**. Then the above equation says that the value of what the consumer buys must equal the value of what she sells, which seems sensible enough.

We could also express the budget line when the endowment is present in a form similar to the way we described it before. Now it takes two equations:

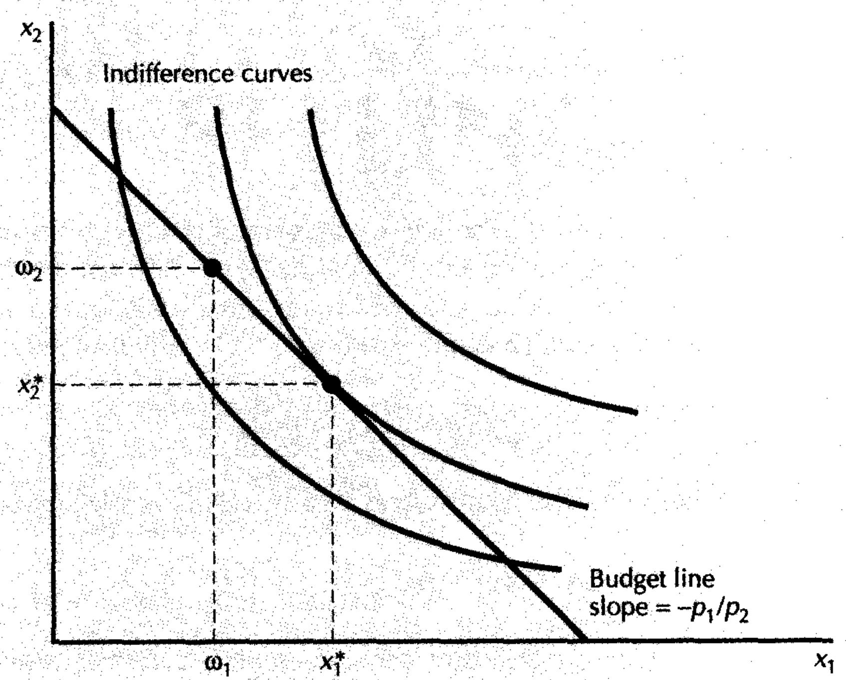
$$\begin{aligned} p_1 x_1 + p_2 x_2 &= m \\ m &= p_1 \omega_1 + p_2 \omega_2. \end{aligned}$$

Once the prices are fixed, the value of the endowment, and hence the consumer's money income, is fixed.

What does the budget line look like graphically? When we fix the prices, money income is fixed, and we have a budget equation just like we had before. Thus the slope must be given by $-p_1/p_2$, just as before, so the only problem is to determine the location of the line.

The location of the line can be determined by the following simple observation: the endowment bundle is always on the budget line. That is, one value of (x_1, x_2) that satisfies the budget line is $x_1 = \omega_1$ and $x_2 = \omega_2$. The endowment is always just affordable, since the amount you have to spend is precisely the value of the endowment.

Putting these facts together shows that the budget line has a slope of $-p_1/p_2$ and passes through the endowment point. This is depicted in Figure 9.1.



The budget line. The budget line passes through the endowment and has a slope of $-p_1/p_2$.

Given this budget constraint, the consumer can choose the optimal consumption bundle just as before. In Figure 9.1 we have shown an example of an optimal consumption bundle (x_1^*, x_2^*). Just as before, it will satisfy the optimality condition that the marginal rate of substitution is equal to the price ratio.

In this particular case, $x_1^* > \omega_1$ and $x_2^* < \omega_2$, so the consumer is a net buyer of good 1 and a net seller of good 2. The net demands are simply the net amounts that the consumer buys or sells of the two goods. In general the consumer may decide to be either a buyer or a seller depending on the relative prices of the two goods.

9.3 Changing the Endowment

In our previous analysis of choice we examined how the optimal consumption changed as the money income changed while the prices remained fixed. We can do a similar analysis here by asking how the optimal consumption changes as the *endowment* changes while the prices remain fixed.

For example, suppose that the endowment changes from (ω_1, ω_2) to some other value (ω_1', ω_2') such that

$$p_1\omega_1 + p_2\omega_2 > p_1\omega_1' + p_2\omega_2'.$$

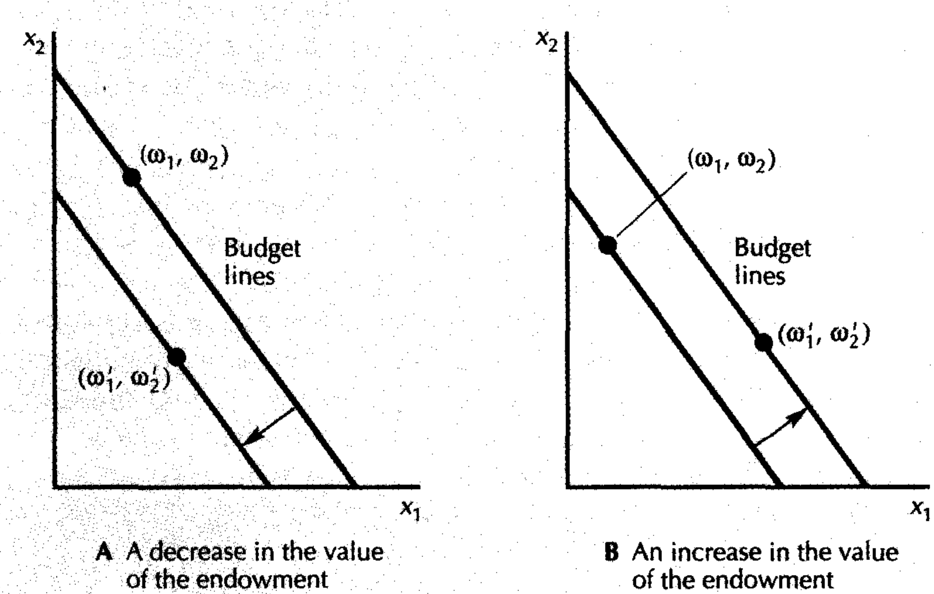
This inequality means that the new endowment (ω_1', ω_2') is worth less than the old endowment—the money income that the consumer could achieve by selling her endowment is less.

This is depicted graphically in Figure 9.2A: the budget line shifts inward. Since this is exactly the same as a reduction in money income, we can conclude the same two things that we concluded in our examination of that case. First, the consumer is definitely worse off with the endowment (ω_1', ω_2') than she was with the old endowment, since her consumption possibilities have been reduced. Second, her demand for each good will change according to whether that good is a normal good or an inferior good.

For example, if good 1 is a normal good and the consumer's endowment changes in a way that reduces its value, we can conclude that the consumer's demand for good 1 will decrease.

The case where the value of the endowment increases is depicted in Figure 9.2B. Following the above argument we conclude that if the budget line shifts outward in a parallel way, the consumer must be made better off. Algebraically, if the endowment changes from (ω_1, ω_2) to (ω_1', ω_2') and $p_1\omega_1 + p_2\omega_2 < p_1\omega_1' + p_2\omega_2'$, then the consumer's new budget set must contain her old budget set. This in turn implies that the optimal choice of the consumer with the new budget set must be preferred to the optimal choice given the old endowment.

It is worthwhile pondering this point a moment. In Chapter 7 we argued that just because a consumption bundle had a higher cost than another didn't mean that it would be preferred to the other bundle. But that only holds for a bundle that must be *consumed*. If a consumer can sell a bundle of goods on a free market at constant prices, then she will always prefer a higher-valued bundle to a lower-valued bundle, simply because a



Changes in the value of the endowment. In case A the value of the endowment decreases, and in case B it increases.

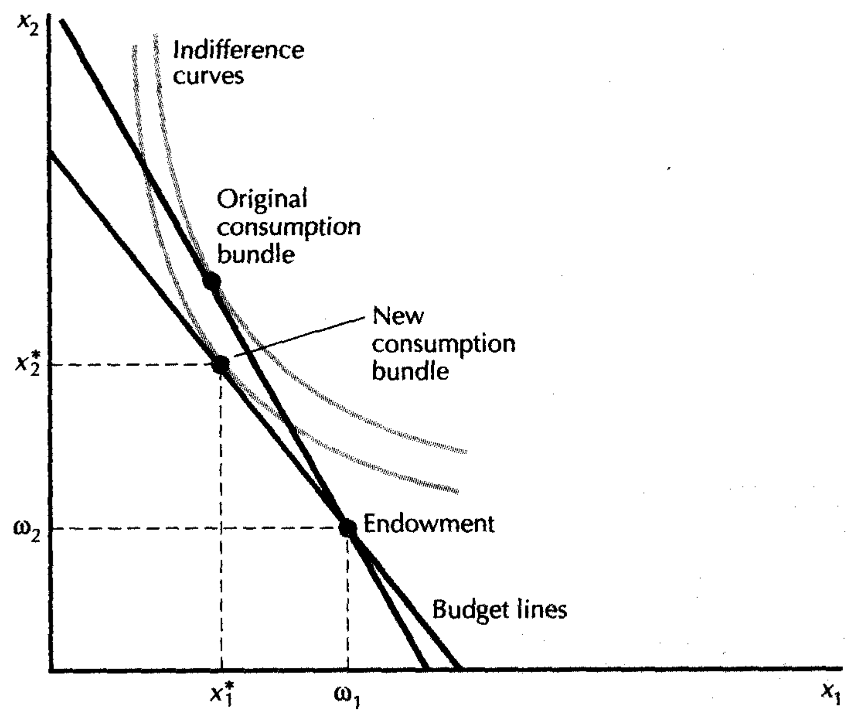
higher-valued bundle gives her more income, and thus more consumption possibilities. Therefore, an *endowment* that has a higher value will always be preferred to an endowment with a lower value. This simple observation will turn out to have some important implications later on.

There's one more case to consider: what happens if $p_1\omega_1 + p_2\omega_2 = p_1\omega_1' + p_2\omega_2'$? Then the budget set doesn't change at all: the consumer is just as well-off with (ω_1, ω_2) as with (ω_1', ω_2'), and her optimal choice should be exactly the same. The endowment has just shifted along the original budget line.

9.4 Price Changes

Earlier, when we examined how demand changed when price changed, we conducted our investigation under the hypothesis that money income remained constant. Now, when money income is determined by the value of the endowment, such a hypothesis is unreasonable: if the value of a good you are selling changes, your money income will certainly change. Thus in the case where the consumer has an endowment, changing prices automatically implies changing income.

Let us first think about this geometrically. If the price of good 1 decreases, we know that the budget line becomes flatter. Since the endowment bundle is always affordable, this means that the budget line must pivot around the endowment, as depicted in Figure 9.3.



Decreasing the price of good 1. Lowering the price of good 1 makes the budget line pivot around the endowment. If the consumer remains a supplier she must be worse off.

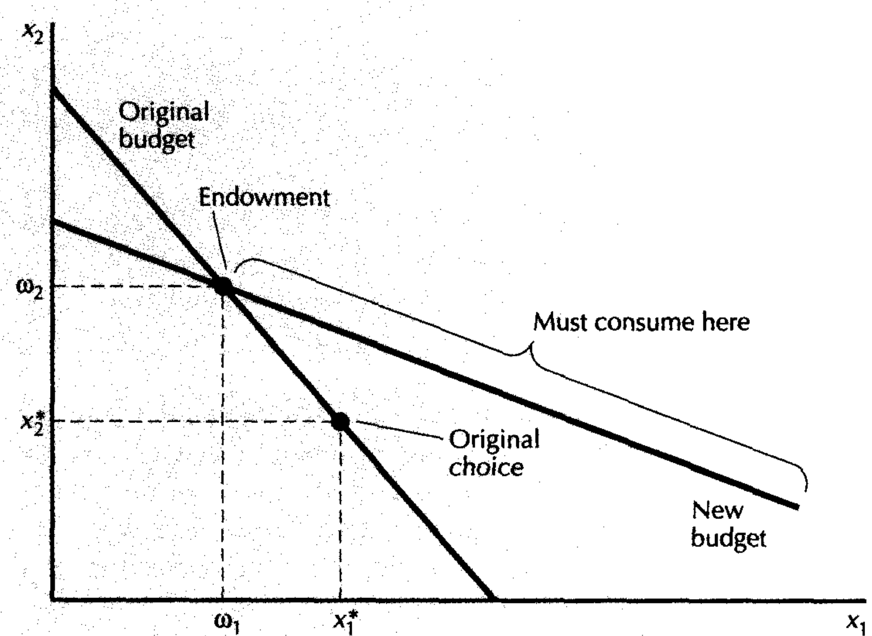
In this case, the consumer is initially a seller of good 1 and remains a seller of good 1 even after the price has *declined.* What can we say about this consumer's welfare? In the case depicted, the consumer is on a lower indifference curve after the price change than before, but will this be true in general? The answer comes from applying the principle of revealed preference.

If the consumer remains a supplier, then her new consumption bundle must be on the colored part of the new budget line. But this part of the new budget line is inside the original budget set: all of these choices were open to the consumer before the price changed. Therefore, by revealed preference, all of these choices are worse than the original consumption bundle. We can therefore conclude that if the price of a good that a consumer is selling goes down, and the consumer decides to remain a seller, then the consumer's welfare must have declined.

What if the price of a good that the consumer is selling decreases and the consumer decides to switch to being a buyer of that good? In this case, the consumer may be better off or she may be worse off—there is no way to tell.

Let us now turn to the situation where the consumer is a net buyer of a good. In this case everything neatly turns around: if the consumer is a net buyer of a good, its price *increases*, and the consumer optimally decides to remain a buyer, then she must definitely be worse off. But if the price increase leads her to become a seller, it could go either way—she may be better off, or she may be worse off. These observations follow from a simple application of revealed preference just like the cases described above, but it is good practice for you to draw a graph just to make sure you understand how this works.

Revealed preference also allows us to make some interesting points about the decision of whether to remain a buyer or to become a seller when prices change. Suppose, as in Figure 9.4, that the consumer is a net buyer of good 1, and consider what happens if the price of good 1 *decreases*. Then the budget line becomes flatter as in Figure 9.4.



Decreasing the price of good 1. If a person is a buyer and the price of what she is buying decreases, she remains a buyer.

As usual we don't know for certain whether the consumer will buy more or less of good 1—it depends on her tastes. However, we can say something for sure: *the consumer will continue to be a net buyer of good 1—she will not switch to being a seller.*

How do we know this? Well, consider what would happen if the consumer did switch. Then she would be consuming somewhere on the colored part of the new budget line in Figure 9.4. But those consumption bundles were feasible for her when she faced the original budget line, and she rejected them in favor of (x_1^*, x_2^*). So (x_1^*, x_2^*) must be better than any of those points. And under the *new* budget line, (x_1^*, x_2^*) is a feasible consumption bundle. So whatever she consumes under the new budget line, it must be better than (x_1^*, x_2^*)—and thus better than any points on the colored part of the new budget line. This implies that her consumption of x_1 must be to the right of her endowment point—that is, she must remain a net demander of good 1.

Again, this kind of observation applies equally well to a person who is a net seller of a good: if the price of what she is selling goes *up*, she will not switch to being a net buyer. We can't tell for sure if the consumer will consume more or less of the good she is selling—but we know that she will keep selling it if the price goes up.

9.5 Offer Curves and Demand Curves

Recall from Chapter 6 that price offer curves depict those combinations of both goods that may be demanded by a consumer and that demand curves depict the relationship between the price and the quantity demanded of some good. Exactly the same constructions work when the consumer has an endowment of both goods.

Consider, for example, Figure 9.5, which illustrates the price offer curve and the demand curve for a consumer. The offer curve will always pass through the endowment, because at some price the endowment will be a demanded bundle; that is, at some prices the consumer will optimally choose not to trade.

As we've seen, the consumer may decide to be a buyer of good 1 for some prices and a seller of good 1 for other prices. Thus the offer curve will generally pass to the left and to the right of the endowment point.

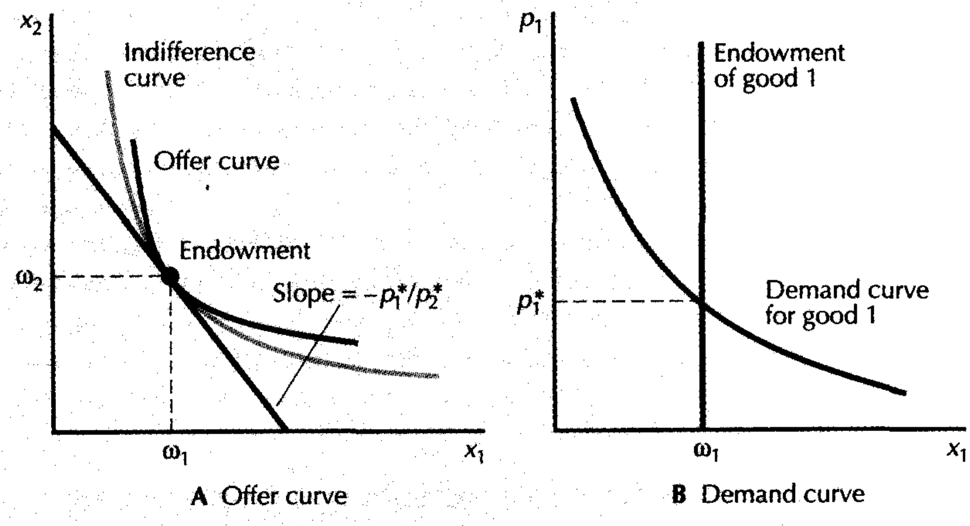
The demand curve illustrated in Figure 9.5B is the gross demand curve—it measures the total amount the consumer chooses to consume of good 1. We have illustrated the net demand curve in Figure 9.6.

Note that the net demand for good 1 will typically be negative for some prices. This will be when the price of good 1 becomes so high that the consumer chooses to become a seller of good 1. At some price the consumer switches between being a net demander to being a net supplier of good 1.

It is conventional to plot the supply curve in the positive orthant, although it actually makes more sense to think of supply as just a negative demand. We'll bow to tradition here and plot the net supply curve in the normal way—as a positive amount, as in Figure 9.6.

Algebraically the net demand for good 1, $d_1(p_1, p_2)$, is the difference between the gross demand $x_1(p_1, p_2)$ and the endowment of good 1, when this difference is positive; that is, when the consumer wants more of the good than he or she has:

$$d_1(p_1, p_2) = \begin{cases} x_1(p_1, p_2) - \omega_1 & \text{if this is positive;} \\ 0 & \text{otherwise.} \end{cases}$$



The offer curve and the demand curve. These are two ways of depicting the relationship between the demanded bundle and the prices when an endowment is present.

The net supply curve is the difference between how much the consumer has of good 1 and how much he or she wants when *this* difference is positive:

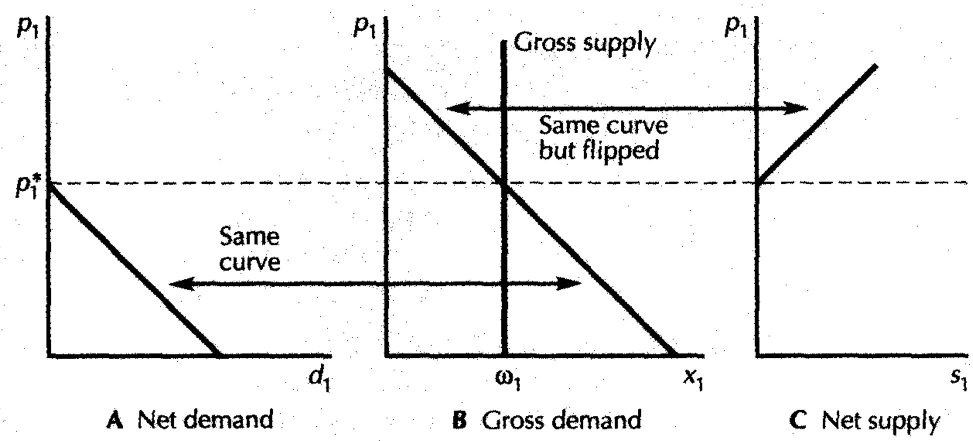
$$s_1(p_1, p_2) = \begin{cases} \omega_1 - x_1(p_1, p_2) & \text{if this is positive;} \\ 0 & \text{otherwise.} \end{cases}$$

Everything that we've established about the properties of demand behavior applies directly to the supply behavior of a consumer—because supply is just negative demand. If the *gross* demand curve is always downward sloping, then the net demand curve will be downward sloping and the supply curve will be upward sloping. Think about it: if an increase in the price makes the net demand more negative, then the net supply will be more positive.

9.6 The Slutsky Equation Revisited

The above applications of revealed preference are handy, but they don't really answer the main question: how does the demand for a good react to a change in its price? We saw in Chapter 8 that if money income was held constant, and the good was a normal good, then a reduction in its price must lead to an increase in demand.

The catch is the phrase "money income was held constant." The case we are examining here necessarily involves a change in money income, since the value of the endowment will necessarily change when a price changes.



Gross demand, net demand, and net supply. Using the gross demand and net demand to depict the demand and supply behavior.

In Chapter 8 we described the Slutsky equation that decomposed the change in demand due to a price change into a substitution effect and an income effect. The income effect was due to the change in purchasing power when prices change. But now, purchasing power has two reasons to change when a price changes. The first is the one involved in the definition of the Slutsky equation: when a price falls, for example, you can buy just as much of a good as you were consuming before and have some extra money left over. Let us refer to this as the **ordinary income effect**. But the second effect is new. When the price of a good changes, it changes the value of your endowment and thus changes your money income. For example, if you are a net supplier of a good, then a fall in its price will reduce your money income directly since you won't be able to sell your endowment for as much money as you could before. We will have the same effects that we had before, plus an extra income effect from the influence of the prices on the value of the endowment bundle. We'll call this the **endowment income effect**.

In the earlier form of the Slutsky equation, the amount of money income you had was fixed. Now we have to worry about how your money income changes as the value of your endowment changes. Thus, when we calculate the effect of a change in price on demand, the Slutsky equation will take the form:

total change in demand = change due to substitution effect + change in demand due to ordinary income effect + change in demand due to endowment income effect.

The first two effects are familiar. As before, let us use Δx_1 to stand for the total change in demand, Δx_1^s to stand for the change in demand due to the substitution effect, and Δx_1^m to stand for the change in demand due to the ordinary income effect. Then we can substitute these terms into the above "verbal equation" to get the Slutsky equation in terms of rates of change:

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - x_1 \frac{\Delta x_1^m}{\Delta m} + \text{endowment income effect.} \tag{9.1}$$

What will the last term look like? We'll derive an explicit expression below, but let us first think about what is involved. When the price of the endowment changes, money income will change, and this change in money income will induce a change in demand. Thus the endowment income effect will consist of two terms:

endowment income effect = change in demand when income changes
× the change in income when price changes. (9.2)

Let's look at the second effect first. Since income is defined to be

$$m = p_1\omega_1 + p_2\omega_2,$$

we have

$$\frac{\Delta m}{\Delta p_1} = \omega_1.$$

This tells us how money income changes when the price of good 1 changes: if you have 10 units of good 1 to sell, and its price goes up by \$1, your money income will go up by \$10.

The first term in equation (9.2) is just how demand changes when income changes. We already have an expression for this: it is $\Delta x_1^m/\Delta m$: the change in demand divided by the change in income. Thus the endowment income effect is given by

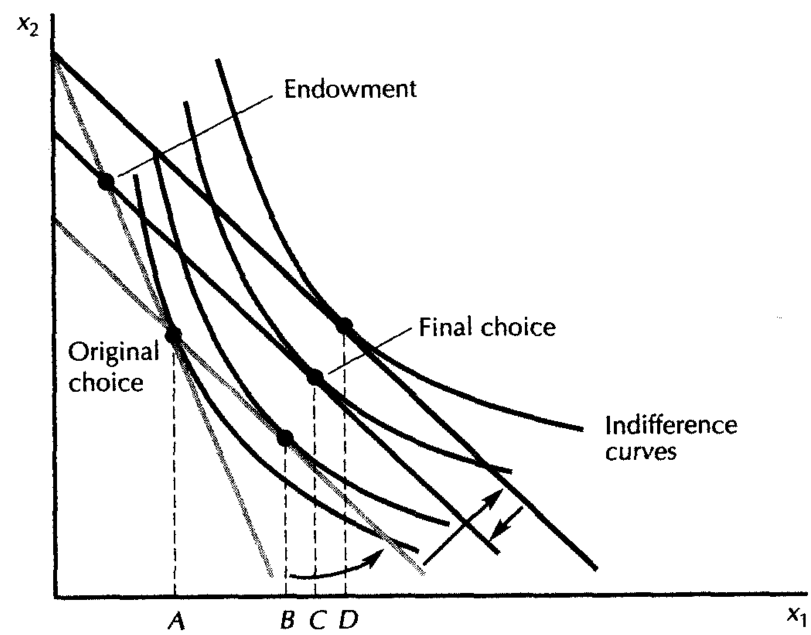
$$\text{endowment income effect} = \frac{\Delta x_1^m}{\Delta m}\frac{\Delta m}{\Delta p_1} = \frac{\Delta x_1^m}{\Delta m}\omega_1. \tag{9.3}$$

Inserting equation (9.3) into equation (9.1) we get the final form of the Slutsky equation:

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} + (\omega_1 - x_1)\frac{\Delta x_1^m}{\Delta m}.$$

This equation can be used to answer the question posed above. We know that the sign of the substitution effect is always negative—opposite the direction of the change in price. Let us suppose that the good is a normal good, so that $\Delta x_1^m/\Delta m > 0$. Then the sign of the combined income effect depends on whether the person is a net demander or a net supplier of the good in question. If the person is a net demander of a normal good, and its price increases, then the consumer will necessarily buy less of it. If the consumer is a net supplier of a normal good, then the sign of the total effect is ambiguous: it depends on the magnitude of the (positive) combined income effect as compared to the magnitude of the (negative) substitution effect.

As before, each of these changes can be depicted graphically, although the graph gets rather messy. Refer to Figure 9.7, which depicts the Slutsky decomposition of a price change. The total change in the demand for good 1 is indicated by the movement from A to C. This is the sum of three separate movements: the substitution effect, which is the movement from A to B, and two income effects. The ordinary income effect, which is the movement from B to D, is the change in demand *holding money income fixed*—that is, the same income effect that we examined in Chapter 8. But since the value of the endowment changes when prices change, there is now an extra income effect: because of the change in the value of the endowment, money income changes. This change in money income shifts the budget line back inward so that it passes through the endowment bundle. The change in demand from D to C measures this endowment income effect.



The Slutsky equation revisited. Breaking up the effect of the price change into the substitution effect (A to B), the ordinary income effect (B to D), and the endowment income effect (D to C).

9.7 Use of the Slutsky Equation

Suppose that we have a consumer who sells apples and oranges that he grows on a few trees in his backyard, like the consumer we described at the beginning of Chapter 8. We said there that if the price of apples increased, then this consumer might actually consume more apples. Using the Slutsky equation derived in this chapter, it is not hard to see why. If we let x_a stand for the consumer's demand for apples, and let p_a be the price of apples, then we know that

$$\frac{\Delta x_a}{\Delta p_a} = \underset{(-)}{\frac{\Delta x_a^s}{\Delta p_a}} + \underset{(+)}{(\omega_a - x_a)} \underset{(+)}{\frac{\Delta x_a^m}{\Delta m}}.$$

This says that the total change in the demand for apples when the price of apples changes is the substitution effect plus the income effect. The substitution effect works in the right direction—increasing the price decreases the demand for apples. But if apples are a normal good for this consumer, the income effect works in the wrong direction. Since the consumer is a net supplier of apples, the increase in the price of apples increases his money income so much that he wants to consume more apples due to the income effect. If the latter term is strong enough to outweigh the substitution effect, we can easily get the "perverse" result.

EXAMPLE: Calculating the Endowment Income Effect

Let's try a little numerical example. Suppose that a dairy farmer produces 40 quarts of milk a week. Initially the price of milk is \$3 a quart. His demand function for milk, for his own consumption, is

$$x_1 = 10 + \frac{m}{10p_1}.$$

Since he is producing 40 quarts at \$3 a quart, his income is \$120 a week. His initial demand for milk is therefore $x_1 = 14$. Now suppose that the price of milk changes to \$2 a quart. His money income will then change to $m' = 2 \times 40 = \$80$, and his demand will be $x_1' = 10 + 80/20 = 14$.

If his money income had remained fixed at $m = \$120$, he would have purchased $x_1 = 10 + 120/10 \times 2 = 16$ quarts of milk at this price. Thus the endowment income effect—the change in his demand due to the change in the value of his endowment—is -2. The substitution effect and the ordinary income effect for this problem were calculated in Chapter 8.

9.8 Labor Supply

Let us apply the idea of an endowment to analyzing a consumer's labor supply decision. The consumer can choose to work a lot and have relatively high consumption, or can choose to work a little and have a small consumption. The amount of consumption and labor will be determined by the interaction of the consumer's preferences and the budget constraint.

The Budget Constraint

Let us suppose that the consumer initially has some money income M that she receives whether she works or not. This might be income from investments or from relatives, for example. We call this amount the consumer's **nonlabor income**. (The consumer could have zero nonlabor income, but we want to allow for the possibility that it is positive.)

Let us use C to indicate the amount of consumption the consumer has, and use p to denote the price of consumption. Then letting w be the wage rate, and L the amount of labor supplied, we have the budget constraint:

$$pC = M + wL.$$

This says that the value of what the consumer consumes must be equal to her nonlabor income plus her labor income.

Let us try to compare the above formulation to the previous examples of budget constraints. The major difference is that we have something that the consumer is choosing—labor supply—on the right-hand side of the equation. We can easily transpose it to the left-hand side to get

$$pC - wL = M.$$

This is better, but we have a minus sign where we normally have a plus sign. How can we remedy this? Let us suppose that there is some maximum amount of labor supply possible—24 hours a day, 7 days a week, or whatever is compatible with the units of measurement we are using. Let $\overline{L}$ denote this amount of labor time. Then adding $w\overline{L}$ to each side and rearranging we have

$$pC + w(\overline{L} - L) = M + w\overline{L}.$$

Let us define $\overline{C} = M/p$, the amount of consumption that the consumer would have if she didn't work at all. That is, $\overline{C}$ is her endowment of consumption, so we write

$$pC + w(\overline{L} - L) = p\overline{C} + w\overline{L}.$$

Now we have an equation very much like those we've seen before. We have two choice variables on the left-hand side and two endowment variables on the right-hand side. The variable $\overline{L}-L$ can be interpreted as the amount of "leisure"—that is, time that isn't labor time. Let us use the variable R (for relaxation!) to denote leisure, so that $R = \overline{L} - L$. Then the total amount of time you have available for leisure is $\overline{R} = \overline{L}$ and the budget constraint becomes

$$pC + wR = p\overline{C} + w\overline{R}.$$

The above equation is formally identical to the very first budget constraint that we wrote in this chapter. However, it has a much more interesting interpretation. It says that the value of a consumer's consumption plus her leisure has to equal the value of her endowment of consumption and her endowment of time, where her endowment of time is valued at her wage rate. The wage rate is not only the price of labor, it is also the price of *leisure.*

After all, if your wage rate is \$10 an hour and you decide to consume an extra hour's leisure, how much does it cost you? The answer is that it costs you \$10 in forgone income—that's the price of that extra hour's consumption of leisure. Economists sometimes say that the wage rate is the **opportunity cost** of leisure.

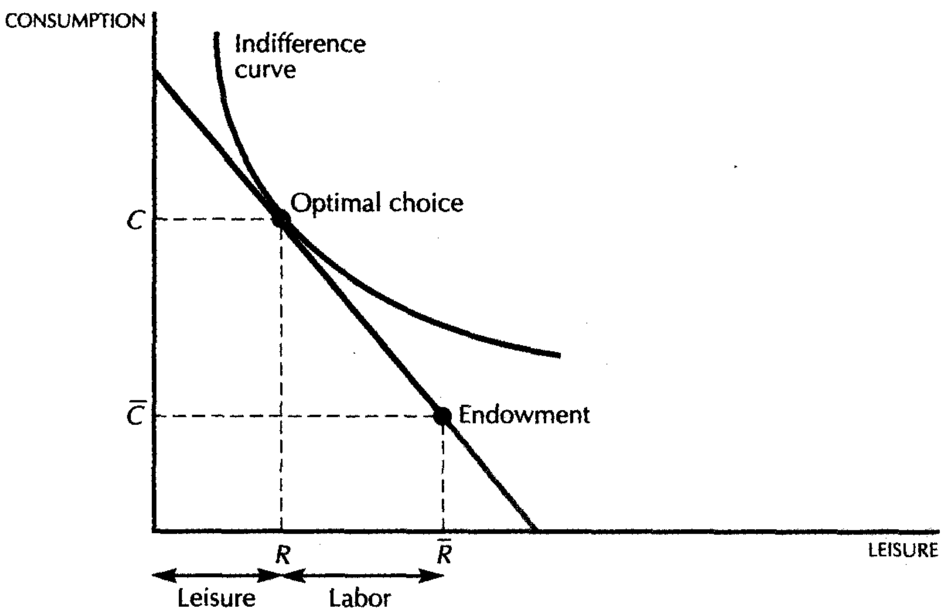
The right-hand side of this budget constraint is sometimes called the consumer's **full income** or **implicit income**. It measures the value of what the consumer owns—her endowment of consumption goods, if any, and her endowment of her own time. This is to be distinguished from the consumer's **measured income**, which is simply the income she receives from selling off some of her time.

The nice thing about this budget constraint is that it is just like the ones we've seen before. It passes through the endowment point $(\overline{L}, \overline{C})$ and has a slope of $-w/p$. The endowment would be what the consumer would get if she did not engage in market trade at all, and the slope of the budget line tells us the rate at which the market will exchange one good for another.

The optimal choice occurs where the marginal rate of substitution—the tradeoff between consumption and leisure—equals w/p, the **real wage**, as depicted in Figure 9.8. The value of the extra consumption to the consumer from working a little more has to be just equal to the value of the lost leisure that it takes to generate that consumption. The real wage is the amount of consumption that the consumer can purchase if she gives up an hour of leisure.

9.9 Comparative Statics of Labor Supply

First let us consider how a consumer's labor supply changes as money income changes with the price and wage held fixed. If you won the state



Labor supply. The optimal choice describes the demand for leisure measured from the origin to the right, and the supply of labor measured from the endowment to the left.

lottery and got a big increase in nonlabor income, what would happen to your supply of labor? What would happen to your demand for leisure?

For most people, the supply of labor would drop when their money income increased. In other words, leisure is probably a normal good for most people: when their money income rises, people choose to consume more leisure. There seems to be a fair amount of evidence for this observation, so we will adopt it as a maintained hypothesis: we will assume that leisure is a normal good.

What does this imply about the response of the consumer's labor supply to changes in the wage rate? When the wage rate increases there are two effects: the return to working more increase and the cost of consuming leisure increases. By using the ideas of income and substitution effects and the Slutsky equation we can isolate these individual effects and analyze them.

When the wage rate increases, leisure becomes more expensive, which by itself leads people to want less of it (the substitution effect). Since leisure is a normal good, we would then predict that an increase in the wage rate would necessarily lead to a decrease in the demand for leisure—that is, an increase in the supply of labor. This follows from the Slutsky equation given in Chapter 8. A normal good must have a negatively sloped demand curve. If leisure is a normal good, then the supply curve of labor must be positively sloped.

But there is a problem with this analysis. First, at an intuitive level, it does not seem reasonable that increasing the wage would *always* result in an increased supply of labor. If my wage becomes very high, I might well "spend" the extra income in consuming leisure. How can we reconcile this apparently plausible behavior with the economic theory given above?

If the theory gives the wrong answer, it is probably because we've misapplied the theory. And indeed in this case we have. The Slutsky example described earlier gave the change in demand *holding money income constant.* But if the wage rate changes, then money income must change as well. The change in demand resulting from a change in money income is an extra income effect—the endowment income effect. It occurs on top of the ordinary income effect.

If we apply the *appropriate* version of the Slutsky equation given earlier in this chapter, we get the following expression:

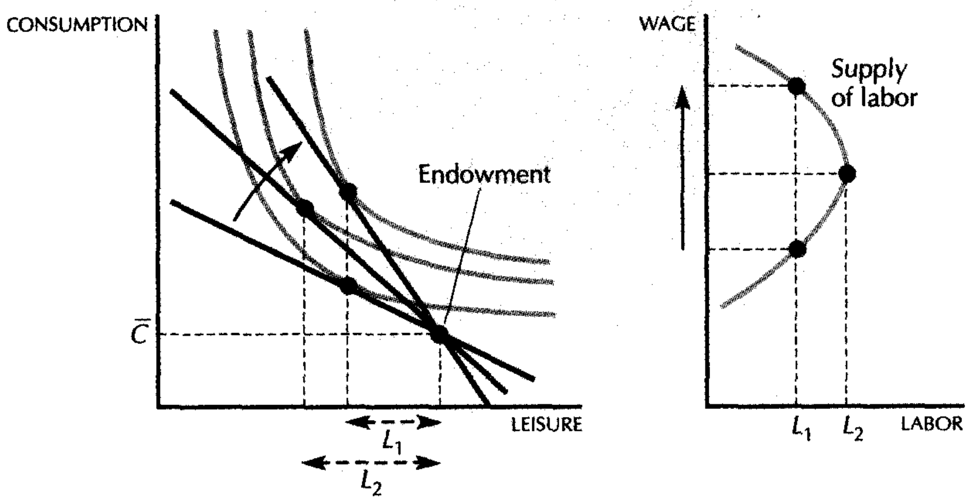
$$\frac{\Delta R}{\Delta w} = \underset{(-)}{\text{substitution effect}} + \underset{(+)}{(\overline{R} - R)} \underset{(+)}{\frac{\Delta R}{\Delta m}}. \tag{9.4}$$

In this expression the substitution effect is definitely negative, as it always is, and $\Delta R/\Delta m$ is positive since we are assuming that leisure is a normal good. But $(\overline{R} - R)$ is positive as well, so the sign of the whole expression is ambiguous. Unlike the usual case of consumer demand, the demand for leisure will have an ambiguous sign, even if leisure is a normal good. As the wage rate increases, people may work more or less.

Why does this ambiguity arise? When the wage rate increases, the substitution effect says work more in order to substitute consumption for leisure. But when the wage rate increases, the value of the endowment goes up as well. This is just like extra income, which may very well be consumed in taking extra leisure. Which is the larger effect is an empirical matter and cannot be decided by theory alone. We have to look at people's actual labor supply decisions to determine which effect dominates.

The case where an increase in the wage rate results in a decrease in the supply of labor is represented by a **backward-bending labor supply curve**. The Slutsky equation tells us that this effect is more likely to occur the larger is $(\overline{R} - R)$, that is, the larger is the supply of labor. When $\overline{R} = R$, the consumer is consuming only leisure, so an increase in the wage will result in a pure substitution effect and thus an increase in the supply of labor. But as the labor supply increases, each increase in the wage gives the consumer additional income for all the hours he is working, so that after some point he may well decide to use this extra income to "purchase" additional leisure—that is, to *reduce* his supply of labor.

A backward-bending labor supply curve is depicted in Figure 9.9. When the wage rate is small, the substitution effect is larger than the income effect, and an increase in the wage will decrease the demand for leisure and hence increase the supply of labor. But for larger wage rates the income effect may outweigh the substitution effect, and an increase in the wage will *reduce* the supply of labor.



A Indifference curves **B** Labor supply curve

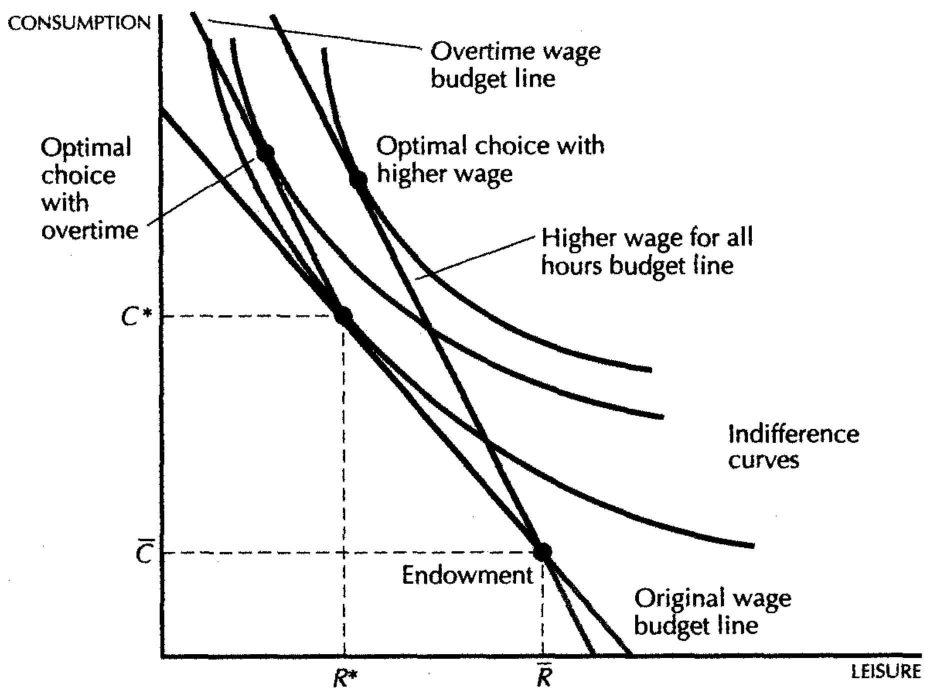
Backward-bending labor supply. As the wage rate increases, the supply of labor increases from L_1 to L_2. But a further increase in the wage rate reduces the supply of labor back to L_1.

EXAMPLE: Overtime and the Supply of Labor

Consider a worker who has chosen to supply a certain amount of labor $L^* = \overline{R} - R^*$ when faced with the wage rate w as depicted in Figure 9.10. Now suppose that the firm offers him a higher wage, $w' > w$, for extra time that he chooses to work. Such a payment is known as an **overtime wage**.

In terms of Figure 9.10, this means that the slope of the budget line will be steeper for labor supplied in excess of L^*. But then we know that the worker will optimally choose to supply more labor, by the usual sort of revealed preference argument: the choices involving working less than L^* were available before the overtime was offered and were rejected.

Note that we get an unambiguous increase in labor supply with an overtime wage, whereas just offering a higher wage for all hours worked has an ambiguous effect—as discussed above, labor supply may increase or it may decrease. The reason is that the response to an overtime wage is essentially a pure substitution effect—the change in the optimal choice resulting from



Overtime versus an ordinary wage increase. An increase in the overtime wage definitely increases the supply of labor, while an increase in the straight wage could decrease the supply of labor.

pivoting the budget line around the chosen point. Overtime gives a higher payment for the *extra* hours worked, whereas a straight increase in the wage gives a higher payment for *all* hours worked. Thus a straight-wage increase involves both a substitution and an income effect while an overtime-wage increase results in a pure substitution effect. An example of this is shown in Figure 9.10. There an increase in the straight wage results in a *decrease* in labor supply, while an increase in the overtime wage results in an increase in labor supply.

Summary

1. Consumers earn income by selling their endowment of goods.

2. The gross demand for a good is the amount that the consumer ends up consuming. The net demand for a good is the amount the consumer buys. Thus the net demand is the difference between the gross demand and the endowment.

3. The budget constraint has a slope of $-p_1/p_2$ and passes through the endowment bundle.

4. When a price changes, the value of what the consumer has to sell will change and thereby generate an additional income effect in the Slutsky equation.

5. Labor supply is an interesting example of the interaction of income and substitution effects. Due to the interaction of these two effects, the response of labor supply to a change in the wage rate is ambiguous.

REVIEW QUESTIONS

1. If a consumer's net demands are $(5, -3)$ and her endowment is $(4, 4)$, what are her gross demands?

2. The prices are $(p_1, p_2) = (2, 3)$, and the consumer is currently consuming $(x_1, x_2) = (4, 4)$. There is a perfect market for the two goods in which they can be bought and sold costlessly. Will the consumer necessarily prefer consuming the bundle $(y_1, y_2) = (3, 5)$? Will she necessarily prefer having the bundle (y_1, y_2)?

3. The prices are $(p_1, p_2) = (2, 3)$, and the consumer is currently consuming $(x_1, x_2) = (4, 4)$. Now the prices change to $(q_1, q_2) = (2, 4)$. Could the consumer be better off under these new prices?

4. The U.S. currently imports about half of the petroleum that it uses. The rest of its needs are met by domestic production. Could the price of oil rise so much that the U.S. would be made better off?

5. Suppose that by some miracle the number of hours in the day increased from 24 to 30 hours (with luck this would happen shortly before exam week). How would this affect the budget constraint?

6. If leisure is an inferior good, what can you say about the slope of the labor supply curve?

APPENDIX

The derivation of the Slutsky equation in the text contained one bit of hand waving. When we considered how changing the monetary value of the endowment affects demand, we said that it was equal to $\Delta x_1^m/\Delta m$. In our old version of the Slutsky equation this was the rate of change in demand when income changed so as to keep the original consumption bundle affordable. But that will not necessarily be equal to the rate of change of demand when the value of the endowment changes. Let's examine this point in a little more detail.

Let the price of good 1 change from p_1 to p_1', and use m'' to denote the new money income at the price p_1' due to the change in the value of the endowment. Suppose that the price of good 2 remains fixed so we can omit it as an argument of the demand function.

By definition of m'', we know that

$$m'' - m = \Delta p_1 \omega_1.$$

Note that it is identically true that

$$\frac{x_1(p_1', m'') - x_1(p_1, m)}{\Delta p_1} =$$

$$\begin{aligned}
&+ \frac{x_1(p_1', m') - x_1(p_1, m)}{\Delta p_1} \quad &\text{(substitution effect)}\\
&- \frac{x_1(p_1', m') - x_1(p_1', m)}{\Delta p_1} \quad &\text{(ordinary income effect)}\\
&+ \frac{x_1(p_1', m'') - x_1(p_1', m)}{\Delta p_1} \quad &\text{(endowment income effect).}
\end{aligned}$$

(Just cancel out identical terms with opposite signs on the right-hand side.)

By definition of the ordinary income effect,

$$\Delta p_1 = \frac{m' - m}{x_1}$$

and by definition of the endowment income effect,

$$\Delta p_1 = \frac{m'' - m}{\omega_1}.$$

Making these replacements gives us a Slutsky equation of the form

$$\frac{x_1(p_1', m'') - x_1(p_1, m)}{\Delta p_1} =$$

$$\begin{aligned}
&+ \frac{x_1(p_1', m') - x_1(p_1, m)}{\Delta p_1} \quad &\text{(substitution effect)}\\
&- \frac{x_1(p_1', m') - x_1(p_1', m)}{m' - m} x_1 \quad &\text{(ordinary income effect)}\\
&+ \frac{x_1(p_1', m'') - x_1(p_1', m)}{m'' - m} \omega_1 \quad &\text{(endowment income effect).}
\end{aligned}$$

Writing this in terms of Δs, we have

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} - \frac{\Delta x_1^m}{\Delta m} x_1 + \frac{\Delta x_1^w}{\Delta m} \omega_1.$$

The only new term here is the last one. It tells how the demand for good 1 changes as income changes, times the *endowment* of good 1. This is precisely the endowment income effect.

Suppose that we are considering a very small price change, and thus a small associated income change. Then the fractions in the two income effects will be virtually the same, since the *rate* of change of good 1 when income changes from m to m' should be about the same as when income changes from m to m''. For such small changes we can collect terms and write the last two terms—the income effects—as

$$\frac{\Delta x_1^m}{\Delta m}(\omega_1 - x_1),$$

which yields a Slutsky equation of the same form as that derived earlier:

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^s}{\Delta p_1} + (\omega_1 - x_1)\frac{\Delta x_1^m}{\Delta m}.$$

If we want to express the Slutsky equation in calculus terms, we can just take limits in this expression. Or, if you prefer, we can calculate the correct equation directly, just by taking partial derivatives. Let $x_1(p_1, m(p_1))$ be the demand function for good 1 where we hold price 2 fixed and recognize that money income depends on the price of good 1 via the relationship $m(p_1) = p_1\omega_1 + p_2\omega_2$. Then we can write

$$\frac{dx_1(p_1, m(p_1))}{dp_1} = \frac{\partial x_1(p_1, m)}{\partial p_1} + \frac{\partial x_1(p_1, m)}{\partial m}\frac{dm(p_1)}{dp_1}. \tag{9.5}$$

By the definition of $m(p_1)$ we know how income changes when price changes:

$$\frac{\partial m(p_1)}{\partial p_1} = \omega_1, \tag{9.6}$$

and by the Slutsky equation we know how demand changes when price changes, holding money income fixed:

$$\frac{\partial x_1(p_1, m)}{\partial p_1} = \frac{\partial x_1^s(p_1)}{\partial p_1} - \frac{\partial x(p_1, m)}{\partial m}x_1. \tag{9.7}$$

Inserting equations (9.6) and (9.7) into equation (9.5) we have

$$\frac{dx_1(p_1, m(p_1))}{dp_1} = \frac{\partial x_1^s(p_1)}{\partial p_1} + \frac{\partial x(p_1, m)}{\partial m}(\omega_1 - x_1),$$

which is the form of the Slutsky equation that we want.